COROLLARY 996. For every two atomic filters (with possibly different bases) $\mathcal{A}$ and $\mathcal{B}$ there exists at most one bijective reloid triple from $\mathcal{A}$ to $\mathcal{B}$.

PROOF. Suppose that $f$ and $g$ are two different bijective reloids from $\mathcal{A}$ to $\mathcal{B}$. Then $g^{-1}\circ f$ is not the identity reloid (otherwise $g^{-1}\circ f = \mathrm{id}^{\mathsf{RLD}}_{\mathrm{dom}\, f}$ and so $f = g$). But $g^{-1}\circ f$ is a bijective reloid (as a composition of bijective reloids) from $\mathcal{A}$ to $\mathcal{A}$ what is impossible. □

## 13.3. Rudin-Keisler equivalence and Rudin-Keisler order

THEOREM 997. Atomic filters $a$ and $b$ (with possibly different bases) are isomorphic iff $a \geq b \wedge b \geq a$.

PROOF. Let $a \geq b \wedge b \geq a$. Then there are a monovalued reloids $f$ and $g$ such that $\mathrm{dom}\, f = a$ and $\mathrm{im}\, f = b$ and $\mathrm{dom}\, g = b$ and $\mathrm{im}\, g = a$. Thus $g\circ f$ and $f\circ g$ are monovalued morphisms from $a$ to $a$ and from $b$ to $b$. By the above we have $g\circ f = \mathrm{id}^{\mathsf{RLD}}_a$ and $f\circ g = \mathrm{id}^{\mathsf{RLD}}_b$ so $g = f^{-1}$ and $f^{-1}\circ f = \mathrm{id}^{\mathsf{RLD}}_a$ and $f\circ f^{-1} = \mathrm{id}^{\mathsf{RLD}}_b$. Thus $f$ is an injective monovalued reloid from $a$ to $b$ and thus $a$ and $b$ are isomorphic. □

The last theorem cannot be generalized from atomic filters to arbitrary filters, as it's shown by the following example:

EXAMPLE 998. $\mathcal{A} \geq_1 \mathcal{B} \wedge \mathcal{B} \geq_1 \mathcal{A}$ but $\mathcal{A}$ is not isomorphic to $\mathcal{B}$ for some filters $\mathcal{A}$ and $\mathcal{B}$.

PROOF. Consider $\mathcal{A} = \uparrow^{\mathbb{R}} [0;1]$ and $\mathcal{B} = \bigsqcap\left\{ \frac{\uparrow^{\mathbb{R}}[0;1+\epsilon)}{\epsilon>0} \right\}$. Then the function $f = \lambda x\in\mathbb{R} : x/2$ witnesses both inequalities $\mathcal{A} \geq_1 \mathcal{B}$ and $\mathcal{B} \geq_1 \mathcal{A}$. But these filters cannot be isomorphic because only one of them is principal. □

LEMMA 999. Let $f_0$ and $f_1$ be **Set**-morphisms. Let $f(x;y) = (f_0 x; f_1 y)$ for a function $f$. Then

$$\left\langle \uparrow^{\mathsf{FCD}(\mathrm{Src}\, f_0\times\mathrm{Src}\, f_1;\mathrm{Dst}\, f_0\times\mathrm{Dst}\, f_1)} \right\rangle (\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}) = \langle\uparrow^{\mathsf{FCD}} f_0\rangle\mathcal{A}\times^{\mathsf{RLD}}\langle\uparrow^{\mathsf{FCD}} f_1\rangle\mathcal{B}.$$

PROOF.

$$\begin{aligned}
&\left\langle \uparrow^{\mathsf{FCD}(\mathrm{Src}\, f_0\times\mathrm{Src}\, f_1;\mathrm{Dst}\, f_0\times\mathrm{Dst}\, f_1)} \right\rangle (\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}) = \\
&\left\langle \uparrow^{\mathsf{FCD}(\mathrm{Src}\, f_0\times\mathrm{Src}\, f_1;\mathrm{Dst}\, f_0\times\mathrm{Dst}\, f_1)} \right\rangle \bigsqcap\left\{ \frac{\uparrow^{\mathrm{Src}\, f_0\times\mathrm{Src}\, f_1} (A\times B)}{A\in\mathcal{A}, B\in\mathcal{B}} \right\} = \\
&\bigsqcap\left\{ \frac{\uparrow^{\mathrm{Dst}\, f_0\times\mathrm{Dst}\, f_1} \langle f\rangle^*(A\times B)}{A\in\mathcal{A}, B\in\mathcal{B}} \right\} = \\
&\bigsqcap\left\{ \frac{\uparrow^{\mathrm{Dst}\, f_0\times\mathrm{Dst}\, f_1} (\langle f_0\rangle^* A\times\langle f_1\rangle^* B)}{A\in\mathcal{A}, B\in\mathcal{B}} \right\} = \\
&\bigsqcap\left\{ \frac{\uparrow^{\mathrm{Dst}\, f_0} \langle f_0\rangle^* A\times \uparrow^{\mathrm{Dst}\, f_1} \langle f_1\rangle^* B)}{A\in\mathcal{A}, B\in\mathcal{B}} \right\} = \quad \text{(theorem 646)} \\
&\bigsqcap\left\{ \frac{\uparrow^{\mathrm{Dst}\, f_0} \langle f_0\rangle^* A}{A\in\mathcal{A}} \right\} \times^{\mathsf{RLD}} \bigsqcap\left\{ \frac{\uparrow^{\mathrm{Dst}\, f_1} \langle f_1\rangle^* B}{B\in\mathcal{B}} \right\} = \\
&\langle\uparrow^{\mathsf{FCD}} f_0\rangle\mathcal{A}\times^{\mathsf{RLD}}\langle\uparrow^{\mathsf{FCD}} f_1\rangle\mathcal{B}.
\end{aligned}$$

□

THEOREM 1000. Let $f$ be a monovalued reloid. Then $\mathrm{GR}\, f$ is isomorphic to the filter $\mathrm{dom}\, f$.