Obviously $f = (\uparrow^{\mathsf{RLD}} F)|_{\mathcal{A}}$ is monovalued and injective.

$$\begin{aligned}\operatorname{im} f &= \\ \bigsqcap\left\{ \frac{\uparrow^B \operatorname{im} G}{G \in (\uparrow^{\mathsf{RLD}} F)|_{\mathcal{A}}} \right\} &= \\ \bigsqcap\left\{ \frac{\uparrow^B \operatorname{im}(H \cap F|_X)}{H \in (\uparrow^{\mathsf{RLD}} F)|_{\mathcal{A}}, X \in \mathcal{A}} \right\} &= \\ \bigsqcap\left\{ \frac{\uparrow^B \operatorname{im} F|_P}{P \in \mathcal{A}} \right\} &= \\ \bigsqcap\left\{ \frac{\uparrow^B \langle F\rangle^* P}{P \in \mathcal{A}} \right\} &= \\ \bigsqcap\left\{ \frac{\uparrow^B \langle F\rangle^* P}{P \in \mathscr{P} A \cap \mathcal{A}} \right\} &= \\ \bigsqcap\left\langle \uparrow^B \right\rangle^* (\mathscr{P} B \cap \mathcal{B}) &= \\ \bigsqcap\left\langle \uparrow^B \right\rangle^* \mathcal{B} = \mathcal{B}.\end{aligned}$$

Thus $\operatorname{dom} f = \mathcal{A}$ and $\operatorname{im} f = \mathcal{B}$.

$\Leftarrow$. Let f be a monovalued injective reloid such that $\operatorname{dom} f = \mathcal{A}$ and $\operatorname{im} f = \mathcal{B}$. Then there exist a function F' and an injective binary relation F'' such that $F', F'' \in \operatorname{GR} f$. Thus $F = F' \cap F''$ is an injection such that $F \in \operatorname{GR} f$. The function F is a bijection from $A = \operatorname{dom} F$ to $B = \operatorname{im} F$. The function $\langle F\rangle^*$ is an injection on $\mathscr{P} A \cap \mathcal{A}$ (and moreover on $\mathscr{P} A$). It's simple to show that $\forall X \in \mathscr{P} A \cap \mathcal{A} : \langle F\rangle^* X \in \mathscr{P} B \cap \mathcal{B}$ and similarly

$$\forall Y \in \mathscr{P} B \cap \mathcal{B} : (\langle F\rangle^*)^{-1} Y = \left\langle F^{-1}\right\rangle^* Y \in \mathscr{P} A \cap \mathcal{A}.$$

Thus $\langle F\rangle^*|_{\mathscr{P} A \cap \mathcal{A}}$ is a bijection $\mathscr{P} A \cap \mathcal{A} \to \mathscr{P} B \cap \mathcal{B}$. So filters $\mathcal{A}$ and $\mathcal{B}$ are isomorphic.

□

Proposition 983. $(\geq_1) = (\sqsupseteq) \circ (\geq_2)$ (when we limit to small filters).

Proof. $\mathcal{A} \geq_1 \mathcal{B}$ iff exists a function $f : \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} \sqsubseteq \left\langle \uparrow^{\mathsf{FCD}} f \right\rangle \mathcal{A}$. But $\mathcal{B} \sqsubseteq \left\langle \uparrow^{\mathsf{FCD}} f \right\rangle \mathcal{A}$ is equivalent to $\exists \mathcal{B}' \in \mathfrak{F} : (\mathcal{B}' \sqsupseteq \mathcal{B} \wedge \mathcal{B}' = \left\langle \uparrow^{\mathsf{FCD}} f \right\rangle \mathcal{A})$. So $\mathcal{A} \geq_1 \mathcal{B}$ is equivalent to existence of $\mathcal{B}' \in \mathfrak{F}$ such that $\mathcal{B}' \sqsupseteq \mathcal{B}$ and existence of a function $f : \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B}' = \left\langle \uparrow^{\mathsf{FCD}} f \right\rangle \mathcal{A}$. This is equivalent to $\mathcal{A}\ ((\sqsupseteq) \circ (\geq_2))\ \mathcal{B}$. □

Proposition 984. If a and b are ultrafilters then $b \geq_1 a \Leftrightarrow b \geq_2 a$.

Proof. We need to prove only $b \geq_1 a \Rightarrow b \geq_2 a$. If $b \geq_1 a$ then there exists a monovalued reloid $f : \operatorname{Base}(b) \to \operatorname{Base}(a)$ such that $\operatorname{dom} f = b$ and $\operatorname{im} f \sqsupseteq a$. Then $\operatorname{im} f = \operatorname{im}(\mathsf{FCD}) f \in \{\bot^{\mathfrak{F}(\operatorname{Base}(a))}\} \cup \operatorname{atoms}^{\mathfrak{F}(\operatorname{Base}(a))}$ because $(\mathsf{FCD}) f$ is a monovalued funcoid. So $\operatorname{im} f = a$ (taken into account $a \neq \bot^{\mathfrak{F}(\operatorname{Base}(a))}$) and thus $b \geq_2 a$. □

Corollary 985. For atomic filters $\geq_1$ is the same as $\geq_2$.

Thus I will write simply $\geq$ for atomic filters.

13.2.1. Existence of no more than one monovalued injective reloid for a given pair of ultrafilters.