2°.

$$f \in \mathrm{Mor}_{\mathbf{GreFunc}_2}(\mathcal{A};\mathcal{B}) \Leftrightarrow \mathcal{B} = f * \mathcal{A} \Leftrightarrow \forall C : (C \in \mathcal{B} \Leftrightarrow C \in f * \mathcal{A}) \Leftrightarrow \forall C \in \mathscr{P}\,\mathrm{Base}(\mathcal{B}) : (C \in \mathcal{B} \Leftrightarrow C \in f * \mathcal{A}) \Leftrightarrow \forall C \in \mathscr{P}\,\mathrm{Base}(\mathcal{B}) : (\langle f^{-1}\rangle^* C \in \mathcal{A} \Leftrightarrow C \in \mathcal{B}).$$

□

DEFINITION 954. The directed multigraph **FuncBij** is the directed multigraph got from $\mathbf{GreFunc}_2$ by restricting to only bijective morphisms.

DEFINITION 955. A filter $\mathcal{A}$ is *directly isomorphic* to a filter $\mathcal{B}$ iff there is a morphism $f \in \mathrm{Mor}_{\mathbf{FuncBij}}(\mathcal{A};\mathcal{B})$.

PROPOSITION 956. $f * \mathcal{A} = \langle\uparrow^{\mathsf{FCD}}\rangle\mathcal{A}$ for every **Set**-morphism $f : \mathrm{Base}(\mathcal{A}) \to \mathrm{Base}(\mathcal{B})$.FiXme: Make it the primary definition instead of the trick with $*$.

PROOF. For every set $C \in \mathscr{P}\,\mathrm{Base}(\mathcal{B})$ we have

$$\begin{aligned} C \in f * \mathcal{A} &\Leftrightarrow \\ \langle f^{-1}\rangle^* C \in \mathcal{A} &\Rightarrow \\ \exists K \in \mathcal{A} : \langle f^{-1}\rangle^* C = K &\Rightarrow \\ \exists K \in \mathcal{A} : \langle f\rangle^* \langle f^{-1}\rangle^* C = \langle f\rangle^* K &\Rightarrow \\ \exists K \in \mathcal{A} : C \supseteq \langle f\rangle^* K &\Leftrightarrow \\ \exists K \in \mathcal{A} : C \in \langle\uparrow^{\mathsf{FCD}} f\rangle^* K &\Rightarrow \\ C \in \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}. \end{aligned}$$

So $C \in f * \mathcal{A} \Rightarrow C \in \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}$.

Let now $C \in \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}$. Then $\uparrow^{\mathrm{Base}(\mathcal{A})} \langle f^{-1}\rangle^* C \sqsupseteq \langle\uparrow^{\mathsf{FCD}} f^{-1}\rangle\langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A} \sqsupseteq \mathcal{A}$ and thus $\langle f^{-1}\rangle^* C \in \mathcal{A}$. □

COROLLARY 957. $f \in \mathrm{Mor}_{\mathbf{GreFunc}_1}(\mathcal{A};\mathcal{B}) \Leftrightarrow \mathcal{B} \sqsubseteq \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}$ for every **Set**-morphism from $\mathrm{Base}(\mathcal{A})$ to $\mathrm{Base}(\mathcal{B})$.

COROLLARY 958. $f \in \mathrm{Mor}_{\mathbf{GreFunc}_2}(\mathcal{A};\mathcal{B}) \Leftrightarrow \mathcal{B} = \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}$ for every **Set**-morphism from $\mathrm{Base}(\mathcal{A})$ to $\mathrm{Base}(\mathcal{B})$.

COROLLARY 959. $\mathcal{A} \geq_1 \mathcal{B}$ iff it exists a **Set**-morphism $f : \mathrm{Base}(\mathcal{A}) \to \mathrm{Base}(\mathcal{B})$ such that $\mathcal{B} \sqsubseteq \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}$.

COROLLARY 960. $\mathcal{A} \geq_2 \mathcal{B}$ iff it exists a **Set**-morphism $f : \mathrm{Base}(\mathcal{A}) \to \mathrm{Base}(\mathcal{B})$ such that $\mathcal{B} = \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}$.

PROPOSITION 961. For a bijective **Set**-morphism $f : \mathrm{Base}(\mathcal{A}) \to \mathrm{Base}(\mathcal{B})$ the following are equivalent:

1°. $\mathcal{B} = f * \mathcal{A}$.
2°. $\forall C \in \mathrm{Base}(\mathcal{B}) : (C \in \mathcal{B} \Leftrightarrow \langle f^{-1}\rangle^* C \in \mathcal{A})$.
3°. $\forall C \in \mathrm{Base}(\mathcal{A}) : (C \in \langle f\rangle^* \mathcal{B} \Leftrightarrow C \in \mathcal{A})$.
4°. $\langle\uparrow^{\mathsf{FCD}} f\rangle|_{\mathcal{A}}$ is a bijection from $\mathcal{A}$ to $\mathcal{B}$.
5°. $\langle\uparrow^{\mathsf{FCD}} f\rangle|_{\mathcal{A}}$ is a function onto $\mathcal{B}$.
6°. $\mathcal{B} = \langle\uparrow^{\mathsf{FCD}} f\rangle\mathcal{A}$.
7°. $f \in \mathrm{Mor}_{\mathbf{GreFunc}_2}(\mathcal{A};\mathcal{B})$.
8°. $f \in \mathrm{Mor}_{\mathbf{FuncBij}}(\mathcal{A};\mathcal{B})$.

PROOF.