Suppose for the contrary that A is infinite. Then A contains more than one non-zero points y, z ($y \neq z$). Without loss of generality $y < z$. So we have that $(y; z)$ is not of the form $(y; y)$ nor $(0; y)$ nor $(y; 0)$. Therefore $A \times A$ isn't a subset of Γ. □

12.2. Totally bounded endoreloids

The below is a straightforward generalization of the customary definition of totally bounded sets on uniform spaces (it's proved below that for uniform spaces the below definitions are equivalent).

DEFINITION 920. An endoreloid f is *α-totally bounded* ($\operatorname{totBound}_\alpha(f)$) if every $E \in \operatorname{xyGR} f$ is α-thick.

DEFINITION 921. An endoreloid f is *β-totally bounded* ($\operatorname{totBound}_\beta(f)$) if every $E \in \operatorname{xyGR} f$ is β-thick.

REMARK 922. We could rewrite the above definitions in a more algebraic way like $\operatorname{xyGR} f \subseteq \operatorname{thick}_\alpha$ (with $\operatorname{thick}_\alpha$ would be defined as a set rather than as a predicate), but we don't really need this simplification.

PROPOSITION 923. If an endoreloid is α-totally bounded then it is β-totally bounded.

PROOF. Because $\operatorname{thick}_\alpha(E) \Rightarrow \operatorname{thick}_\beta(E)$. □

PROPOSITION 924. If an endoreloid f is reflexive and $\operatorname{Ob} f$ is finite then f is both α-totally bounded and β-totally bounded.

PROOF. It enough to prove that f is α-totally bounded. Really, every $E \in \operatorname{xyGR} f$ is reflexive. Thus $\{x\} \times \{x\} \subseteq E$ for $x \in \operatorname{Ob} f$ and thus $\left\{ \frac{\{x\}}{x \in \operatorname{Ob} f} \right\}$ is a sought for finite cover of $\operatorname{Ob} f$. □

OBVIOUS 925.

- A principal endoreloid induced by a **Rel**-morphism E is α-totally bounded iff E is α-thick.
- A principal endoreloid induced by a **Rel**-morphism E is β-totally bounded iff E is β-thick.

EXAMPLE 926. There is a β-totally bounded endoreloid which is not α-totally bounded.

PROOF. It follows from the example above and properties of principal endoreloids. □

12.3. Special case of uniform spaces

DEFINITION 927. *Uniform space* is essentially the same as symmetric, reflexive and transitive endoreloid.

EXERCISE 928. Prove that it is essentially the same as the standard definition of a uniform space (see Wikipedia or PlanetMath).

THEOREM 929. Let f be such a endoreloid that $f \circ f^{-1} \sqsubseteq f$. Then f is α-totally bounded iff it is β-totally bounded.

PROOF.

$\Rightarrow$. Proved above.