THEOREM 902. A filter $\mathcal{A}$ is connected regarding a reloid f iff $\mathcal{A}$ is connected regarding every $F \in \left\langle\uparrow^{\mathsf{RLD}}\right\rangle^* \operatorname{xyGR} f$.

PROOF.

$\Rightarrow$. Obvious.

$\Leftarrow$. $\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{RLD}} F$ iff $S(F) = F^0 \sqcup F^1 \sqcup F^2 \sqcup \dots \in \mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}$.

$$S^*(f) = \bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}} S(F)}{F \in \operatorname{xyGR} f}\right\} \sqsupseteq \bigsqcap\left\{\frac{\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}}{F \in \operatorname{xyGR} f}\right\} = \mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}.$$

□

CONJECTURE 903. A filter $\mathcal{A}$ is connected regarding a funcoid f iff $\mathcal{A}$ is connected regarding every $F \in \left\langle\uparrow^{\mathsf{FCD}}\right\rangle^* \operatorname{xyGR} f$.

The above conjecture is open even for the case when $\mathcal{A}$ is a principal filter.

CONJECTURE 904. A filter $\mathcal{A}$ is connected regarding a reloid f iff it is connected regarding the funcoid $(\mathsf{FCD})f$.

The above conjecture is true in the special case of principal filters:

PROPOSITION 905. A filter $\uparrow^{\operatorname{Ob}\mu} A$ (for a set A) is connected regarding an endoreloid f iff it is connected regarding the endofuncoid $(\mathsf{FCD})f$.

PROOF. $\uparrow^{\operatorname{Ob}\mu} A$ is connected regarding a reloid f iff A is connected regarding every $F \in \operatorname{xyGR} f$ that is when (taken into account that connectedness for $\uparrow^{\mathsf{RLD}} F$ is the same as connectedness of $\uparrow^{\mathsf{FCD}} F$)

$$\begin{aligned}
&\forall F \in \operatorname{xyGR} f \forall \mathcal{X}, \mathcal{Y} \in \mathfrak{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathfrak{F}(\operatorname{Ob} f)}\} : (\mathcal{X} \sqcup \mathcal{Y} = \uparrow^{\operatorname{Ob} f} A \Rightarrow \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}) \Leftrightarrow \\
&\forall \mathcal{X}, \mathcal{Y} \in \mathfrak{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathfrak{F}(\operatorname{Ob} f)}\} \forall F \in \operatorname{xyGR} f : (\mathcal{X} \sqcup \mathcal{Y} = \uparrow^{\operatorname{Ob} f} A \Rightarrow \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}) \Leftrightarrow \\
&\forall \mathcal{X}, \mathcal{Y} \in \mathfrak{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathfrak{F}(\operatorname{Ob} f)}\} (\mathcal{X} \sqcup \mathcal{Y} = \uparrow^{\operatorname{Ob} f} A \Rightarrow \forall F \in \operatorname{xyGR} f : \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}) \Leftrightarrow \\
&\qquad \forall \mathcal{X}, \mathcal{Y} \in \mathfrak{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathfrak{F}(\operatorname{Ob} f)}\} (\mathcal{X} \sqcup \mathcal{Y} = \uparrow^{\operatorname{Ob} f} A \Rightarrow \mathcal{X} \left[(\mathsf{FCD})f\right] \mathcal{Y})
\end{aligned}$$

that is when the set $\uparrow^{\operatorname{Ob}\mu} A$ is connected regarding the funcoid $(\mathsf{FCD})f$. □

CONJECTURE 906. A set A is connected regarding an endofuncoid μ iff for every $a, b \in A$ there exists a totally ordered set $P \subseteq A$ such that $\min P = a$, $\max P = b$ and

$$\forall q \in P \setminus \{b\} : \left\{\frac{x \in P}{x \leq q}\right\} [\mu]^* \left\{\frac{x \in P}{x > q}\right\}.$$

Weaker condition:

$$\forall q \in P \setminus \{b\} : \left\{\frac{x \in P}{x \leq q}\right\} [\mu]^* \left\{\frac{x \in P}{x > q}\right\} \vee \forall q \in P \setminus \{a\} : \left\{\frac{x \in P}{x < q}\right\} [\mu]^* \left\{\frac{x \in P}{x \geq q}\right\}.$$

11.5. Algebraic properties of S and S^*

THEOREM 907. $S^*(S^*(f)) = S^*(f)$ for every endoreloid f.