Proposition 892. $S^*(\mu) = \mathrm{id}^{\mathsf{RLD}(\operatorname{Ob}\mu)} \sqcup S_1^*(\mu)$ for every endoreloid μ.

Proof. By the proposition 457. □

Proposition 893. $S^*(\mu) = S(\mu)$ if μ is a principal reloid.

Proof. $S^*(\mu) = \bigsqcap\{S(\mu)\} = S(\mu)$. □

Definition 894. A filter $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$ is called *connected* regarding an endoreloid μ when $S^*(\mu \sqcap (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A})) \sqsupseteq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}$.

Obvious 895. A filter $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$ is connected regarding an endoreloid μ iff $S^*(\mu \sqcap (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A})) = \mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}$.

Definition 896. A filter $\mathcal{A} \in \mathfrak{F}(\operatorname{Ob}\mu)$ is called *connected* regarding an endofuncoid μ when

$$\forall \mathcal{X}, \mathcal{Y} \in \mathfrak{F}(\operatorname{Ob}\mu) \setminus \{\bot^{\mathfrak{F}(\operatorname{Ob}\mu)}\} : (\mathcal{X} \sqcup \mathcal{Y} = \mathcal{A} \Rightarrow \mathcal{X} \ [\mu] \ \mathcal{Y}).$$

Proposition 897. Let A be a set. The filter $\uparrow^{\operatorname{Ob}\mu} A$ is connected regarding an endofuncoid μ iff

$$\forall X, Y \in \mathscr{P}(\operatorname{Ob}\mu) \setminus \{\emptyset\} : (X \cup Y = A \Rightarrow X \ [\mu]^* \ Y).$$

Proof.

$\Rightarrow$. Obvious.

$\Leftarrow$. It follows from co-separability of filters.

□

Theorem 898. The following are equivalent for every set A and binary relation μ on a set U:

1°. A is connected regarding binary relation μ.
2°. $\uparrow^U A$ is connected regarding $\uparrow^{\mathsf{RLD}(U;U)} \mu$.
3°. $\uparrow^U A$ is connected regarding $\uparrow^{\mathsf{FCD}(U;U)} \mu$.

Proof.

1°⇔2°.

$$S^*(\uparrow^{\mathsf{RLD}(U;U)} \mu \sqcap (\uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A)) = S^*(\uparrow^{\mathsf{RLD}(U;U)} (\mu \cap (A \times A))) = \uparrow^{\mathsf{RLD}(U;U)} S(\mu \cap (A \times A)).$$

So

$$S^*(\uparrow^{\mathsf{RLD}(U;U)} \mu \sqcap (\uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A)) \sqsupseteq \uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A \Leftrightarrow \uparrow^{\mathsf{RLD}(U;U)} S(\mu \cap (A \times A)) \sqsupseteq \uparrow^{\mathsf{RLD}(U;U)} (A \times A) = \uparrow^U A \times^{\mathsf{RLD}} \uparrow^U A.$$

1°⇔3°. It follows from the previous proposition.

□

Next is conjectured a statement more strong than the above theorem:

Conjecture 899. Let $\mathcal{A}$ be a filter on a set U and F is a binary relation on U.

$\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{FCD}(U;U)} F$ iff $\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{RLD}(U;U)} F$.

Obvious 900. A filter $\mathcal{A}$ is connected regarding a reloid μ iff it is connected regarding the reloid $\mu \sqcap (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A})$.

Obvious 901. A filter $\mathcal{A}$ is connected regarding a funcoid μ iff it is connected regarding the funcoid $\mu \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{A})$.