9.4. Proof of the main result

THEOREM 855. $(\bigsqcup T) \circ F = \bigsqcup\left\{\frac{G\circ F}{G\in T}\right\}$ for every principal reloid $F = \uparrow^{\mathsf{RLD}(\operatorname{Src} f; \operatorname{Dst} g)} f$ and a set T of reloids from $\operatorname{Dst} F$ to some set V. (In other words principal reloids are co-metacomplete and thus also metacomplete by duality.)

PROOF.

$$\begin{aligned}
\left(\bigsqcup T\right) \circ F &= \\
\bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f; V)} \right\rangle^* \langle \operatorname{dom} \rangle^* \left(\left(\bigsqcup T\right) \otimes F \right) &= \\
\operatorname{dom} \bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f \times V; \mho)} \right\rangle^* \left(\left(\bigsqcup T\right) \otimes F \right) &= \\
\operatorname{dom} \bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f \times V; \mho)} \right\rangle^* \left\{ \frac{G \otimes f}{G \in \operatorname{GR} \bigsqcup T} \right\}&; \\
\bigsqcup \left\{ \frac{G \circ F}{G \in T} \right\} &= \\
\bigsqcup \left\{ \frac{\bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f; V)} \right\rangle^* \langle \operatorname{dom} \rangle^* (G \otimes F)}{G \in T} \right\} &= \\
\bigsqcup \left\{ \frac{\operatorname{dom} \bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f; V)} \right\rangle^* (G \otimes F)}{G \in T} \right\} &= \\
\operatorname{dom} \bigsqcup \left\{ \frac{\bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f; V)} \right\rangle^* (G \otimes F)}{G \in T} \right\}&.
\end{aligned}$$

It's enough to prove

$$\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}(\operatorname{Src} f \times V; \mho)} (G \otimes f)}{G \in \operatorname{GR} \bigsqcup T} \right\} = \bigsqcup \left\{ \frac{\bigsqcap \left\langle \uparrow^{\mathsf{RLD}(\operatorname{Src} f \times V; \mho)} \right\rangle^* (G \otimes F)}{G \in T} \right\}$$

but this is the statement of the lemma. □

9.5. Embedding reloids into funcoids

DEFINITION 856. Let f be a reloid. The funcoid

$$\rho f = \mathsf{FCD}(\mathscr{P}(\operatorname{Src} f \times \operatorname{Src} f); \mathscr{P}(\operatorname{Dst} f \times \operatorname{Dst} f))$$

is defined by the formulas:

$$\langle \rho f \rangle x = f \circ x \quad \text{and} \quad \langle \rho f^{-1} \rangle y = f^{-1} \circ y$$

where x are endoreloids on $\operatorname{Src} f$ and y are endoreloids on $\operatorname{Dst} f$.

PROPOSITION 857. It is really a funcoid (if we equate reloids x and y with corresponding filters on Cartesian products of sets).

PROOF. $y \not\asymp \langle \rho f \rangle x \Leftrightarrow y \not\asymp f \circ x \Leftrightarrow f^{-1} \circ y \not\asymp x \Leftrightarrow \langle \rho f^{-1} \rangle y \not\asymp x$. □

COROLLARY 858. $(\rho f)^{-1} = \rho f^{-1}$.

DEFINITION 859. It can be continued to arbitrary funcoids x having destination $\operatorname{Src} f$ by the formula $\langle \rho^* f \rangle x = \langle \rho f \rangle \operatorname{id}_{\operatorname{Src} f} \circ x = f \circ x$.

PROPOSITION 860. ρ is an injection.

PROOF. Consider $x = \operatorname{id}_{\operatorname{Src} f}$. □

PROPOSITION 861. $\rho(g \circ f) = (\rho g) \circ (\rho f)$.