PROOF. Let $K \in \operatorname{GR} g$. Then for every sets $X \in \mathscr{P} \operatorname{Src} g$, $Y \in \mathscr{P} \operatorname{Dst} g$

$$X\ [K]^*\ Y \Leftrightarrow X\ \left[\uparrow^{\mathsf{FCD}} K\right]^*\ Y \Leftrightarrow X\ \left[(\mathsf{FCD}) \uparrow^{\mathsf{RLD}} K\right]^*\ Y \Leftarrow X\ [(\mathsf{FCD}) g]^*\ Y.$$

Thus $\uparrow^{\mathsf{FCD}} K \sqsupseteq (\mathsf{FCD}) g$ that is $K \in \operatorname{GR}(\mathsf{FCD}) g$. □

THEOREM 830. $g \circ (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) \circ f = \left\langle (\mathsf{FCD}) f^{-1} \right\rangle \mathcal{A} \times^{\mathsf{RLD}} \langle (\mathsf{FCD}) g \rangle \mathcal{B}$ for every reloids f, g and filters $\mathcal{A} \in \mathfrak{F}(\operatorname{Dst} f)$, $\mathcal{B} \in \mathfrak{F}(\operatorname{Src} g)$.FiXme: Similar proposition for funcoids?

PROOF.

$$\begin{aligned}
& g \circ (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) \circ f = \\
& \bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}(\operatorname{Src} f; \operatorname{Dst} g)} (G \circ (A \times B) \circ F)}{F \in \operatorname{GR} f, G \in \operatorname{GR} g, A \in \mathcal{A}, B \in \mathcal{B}} \right\} = \\
& \bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}(\operatorname{Src} f; \operatorname{Dst} g)} (\left\langle F^{-1} \right\rangle^* A \times \langle G \rangle^* B)}{F \in \operatorname{GR} f, G \in \operatorname{GR} g, A \in \mathcal{A}, B \in \mathcal{B}} \right\} = \\
& \bigsqcap \left\{ \frac{\uparrow^{\operatorname{Src} f} \left\langle F^{-1} \right\rangle^* A \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} g} \langle G \rangle^* B)}{F \in \operatorname{GR} f, G \in \operatorname{GR} g, A \in \mathcal{A}, B \in \mathcal{B}} \right\} = \\
& \text{(theorem 753)} \\
& \bigsqcap \left\{ \frac{\uparrow^{\operatorname{Src} f} \left\langle F^{-1} \right\rangle^* A}{F \in \operatorname{GR} f, A \in \mathcal{A}} \right\} \times^{\mathsf{RLD}} \bigsqcap \left\{ \frac{\uparrow^{\operatorname{Dst} g} \langle G \rangle^* B}{G \in \operatorname{GR} g, B \in \mathcal{B}} \right\} = \\
& \bigsqcap \left\{ \frac{\left\langle \uparrow^{\mathsf{FCD}(\operatorname{Dst} f; \operatorname{Src} f)} F^{-1} \right\rangle^* A}{F \in \operatorname{GR} f, A \in \mathcal{A}} \right\} \times^{\mathsf{RLD}} \bigsqcap \left\{ \frac{\left\langle \uparrow^{\mathsf{FCD}(\operatorname{Src} g; \operatorname{Dst} g)} G \right\rangle^* B}{G \in \operatorname{GR} g, B \in \mathcal{B}} \right\} = \\
& \bigsqcap \left\{ \frac{\left\langle \uparrow^{\mathsf{FCD}(\operatorname{Dst} f; \operatorname{Src} f)} F^{-1} \right\rangle \mathcal{A}}{F \in \operatorname{GR} f} \right\} \times^{\mathsf{RLD}} \bigsqcap \left\{ \frac{\left\langle \uparrow^{\mathsf{FCD}(\operatorname{Src} g; \operatorname{Dst} g)} G \right\rangle \mathcal{B}}{G \in \operatorname{GR} g} \right\} = \\
& \text{(by definition of } (\mathsf{FCD})) \\
& \left\langle (\mathsf{FCD}) f^{-1} \right\rangle \mathcal{A} \times^{\mathsf{RLD}} \langle (\mathsf{FCD}) g \rangle \mathcal{B}.
\end{aligned}$$

□

COROLLARY 831.

1°. $(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) \circ f = \left\langle (\mathsf{FCD}) f^{-1} \right\rangle \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$;
2°. $g \circ (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) = \mathcal{A} \times^{\mathsf{RLD}} \langle (\mathsf{FCD}) g \rangle \mathcal{B}$.

8.3. Galois connections between funcoids and reloids

THEOREM 832. $(\mathsf{FCD}) : \mathsf{RLD}(A; B) \to \mathsf{FCD}(A; B)$ is the lower adjoint of $(\mathsf{RLD})_{\mathrm{in}} : \mathsf{FCD}(A; B) \to \mathsf{RLD}(A; B)$ for every sets A, B.

PROOF. Because (FCD) and $(\mathsf{RLD})_{\mathrm{in}}$ are trivially monotone, it's enough to prove (for every $f \in \mathsf{RLD}(A; B)$, $g \in \mathsf{FCD}(A; B)$)

$$f \sqsubseteq (\mathsf{RLD})_{\mathrm{in}} (\mathsf{FCD}) f \quad \text{and} \quad (\mathsf{FCD}) (\mathsf{RLD})_{\mathrm{in}} g \sqsubseteq g.$$