Further

$$\forall A \in \mathcal{A}, B \in \mathcal{B} : \mathcal{X} \left[\uparrow^{\mathsf{FCD}(\mathrm{Base}(\mathcal{A});\mathrm{Base}(\mathcal{B}))} (A \times B) \right] \mathcal{Y} \Leftrightarrow$$
$$\forall A \in \mathcal{A}, B \in \mathcal{B} : (\mathcal{X} \not\asymp \uparrow^{\mathrm{Base}(\mathcal{A})} A \wedge \mathcal{Y} \not\asymp \uparrow^{\mathrm{Base}(\mathcal{B})} B) \Leftrightarrow$$
$$\mathcal{X} \not\asymp \mathcal{A} \wedge \mathcal{Y} \not\asymp \mathcal{B} \Leftrightarrow \mathcal{X} \left[\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right] \mathcal{Y}.$$

Thus $\mathcal{X} \left[(\mathsf{FCD})(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) \right] \mathcal{Y} \Leftrightarrow \mathcal{X} \left[\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \right] \mathcal{Y}$. □

PROPOSITION 808. $\operatorname{dom}(\mathsf{FCD})f = \operatorname{dom} f$ and $\operatorname{im}(\mathsf{FCD})f = \operatorname{im} f$ for every reloid f.

PROOF.

$$\operatorname{im}(\mathsf{FCD})f = \langle (\mathsf{FCD})f \rangle \top^{\mathfrak{F}(\operatorname{Src} f)} =$$
$$\bigsqcap \left\{ \frac{\uparrow^{\operatorname{Dst} f} \langle F \rangle^* (\operatorname{Src} f)}{F \in \operatorname{GR} f} \right\} =$$
$$\bigsqcap \left\{ \frac{\uparrow^{\operatorname{Dst} f} \operatorname{im} F}{F \in \operatorname{GR} f} \right\} =$$
$$\bigsqcap \langle \uparrow^{\operatorname{Dst} f} \rangle^* \langle \operatorname{im} \rangle^* \operatorname{GR} f = \operatorname{im} f.$$

$\operatorname{dom}(\mathsf{FCD})f = \operatorname{dom} f$ is similar. □

PROPOSITION 809. $(\mathsf{FCD})(f \sqcap (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})) = (\mathsf{FCD})f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B})$ for every reloid f and $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$ and $\mathcal{B} \in \mathfrak{F}(\operatorname{Dst} f)$.

PROOF.

$$(\mathsf{FCD})(f \sqcap (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})) =$$
$$(\mathsf{FCD})(\operatorname{id}^{\mathsf{RLD}}_{\mathcal{B}} \circ f \circ \operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}}) =$$
$$(\mathsf{FCD}) \operatorname{id}^{\mathsf{RLD}}_{\mathcal{B}} \circ (\mathsf{FCD})f \circ (\mathsf{FCD}) \operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}} =$$
$$\operatorname{id}^{\mathsf{FCD}}_{\mathcal{B}} \circ (\mathsf{FCD})f \circ \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}} =$$
$$(\mathsf{FCD})f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}).$$

□

COROLLARY 810. $(\mathsf{FCD})(f|_{\mathcal{A}}) = ((\mathsf{FCD})f)|_{\mathcal{A}}$ for every reloid f and a filter $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$.

PROPOSITION 811. $\langle (\mathsf{FCD})f \rangle \mathcal{X} = \operatorname{im}(f|_{\mathcal{X}})$ for every reloid f and a filter $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$.

PROOF. $\operatorname{im}(f|_{\mathcal{X}}) = \operatorname{im}(\mathsf{FCD})(f|_{\mathcal{X}}) = \operatorname{im}(((\mathsf{FCD})f)|_{\mathcal{X}}) = \langle (\mathsf{FCD})f \rangle \mathcal{X}$. □

PROPOSITION 812. $(\mathsf{FCD})f = \bigsqcup \left\{ \frac{x \times^{\mathsf{FCD}} y}{x \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}, y \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}, x \times^{\mathsf{RLD}} y \not\asymp f} \right\}$ for every reloid f.

PROOF. $(\mathsf{FCD})f = \bigsqcup \left\{ \frac{x \times^{\mathsf{FCD}} y}{x \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}, y \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)}, x \times^{\mathsf{FCD}} y \not\asymp (\mathsf{FCD})f} \right\}$, but $x \times^{\mathsf{FCD}} y \not\asymp (\mathsf{FCD})f \Leftrightarrow x \left[(\mathsf{FCD})f \right] y \Leftrightarrow x \times^{\mathsf{RLD}} y \not\asymp f$, thus follows the theorem. □