So continuing the above equalities,

$$\langle((\mathsf{FCD})g) \circ ((\mathsf{FCD})f)\rangle^* X =$$

$$\bigsqcap\left\{ \frac{\bigsqcap\left\{ \frac{\uparrow^{\operatorname{Dst} g}\langle G\rangle^* \langle F\rangle^* X}{F \in \operatorname{xyGR} f} \right\}}{G \in \operatorname{xyGR} g} \right\} =$$

$$\bigsqcap\left\{ \frac{\bigsqcap \uparrow^{\operatorname{Dst} g} \langle G\rangle^* \langle F\rangle^* X}{F \in \operatorname{xyGR} f, G \in \operatorname{xyGR} g} \right\} =$$

$$\bigsqcap\left\{ \frac{\bigsqcap \uparrow^{\operatorname{Dst} g} \langle G \circ F\rangle^* X}{F \in \operatorname{xyGR} f, G \in \operatorname{xyGR} g} \right\}.$$

Combining these equalities we get $\langle(\mathsf{FCD})(g \circ f)\rangle^* X = \langle((\mathsf{FCD})g) \circ ((\mathsf{FCD})f)\rangle^* X$ for every set $X \in \mathscr{P}(\operatorname{Src} f)$. □

Proposition 805. $(\mathsf{FCD}) \operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}} = \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}}$ for every filter $\mathcal{A}$.

Proof. Recall that $\operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}} = \bigsqcap\left\{ \frac{\uparrow^{\operatorname{Base}(\mathcal{A})} \operatorname{id}_A}{A \in \mathcal{A}} \right\}$. For every $\mathcal{X}, \mathcal{Y} \in \mathfrak{F}(\operatorname{Base}(\mathcal{A}))$we have

$$\begin{aligned}
\mathcal{X} \left[(\mathsf{FCD}) \operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}}\right] \mathcal{Y} &\Leftrightarrow \\
\mathcal{X} \times^{\mathsf{RLD}} \mathcal{Y} \not\asymp \operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}} &\Leftrightarrow \\
\forall A \in \mathcal{A} : \mathcal{X} \times^{\mathsf{RLD}} \mathcal{Y} \not\asymp \uparrow^{\mathsf{RLD}(\operatorname{Base}(\mathcal{A}); \operatorname{Base}(\mathcal{A}))} \operatorname{id}_A &\Leftrightarrow \\
\forall A \in \mathcal{A} : \mathcal{X} \left[\uparrow^{\mathsf{FCD}(\operatorname{Base}(\mathcal{A}); \operatorname{Base}(\mathcal{A}))} \operatorname{id}_A\right] \mathcal{Y} &\Leftrightarrow \\
\forall A \in \mathcal{A} : \mathcal{X} \sqcap \mathcal{Y} \not\asymp \uparrow^{\operatorname{Base}(\mathcal{A})} A &\Leftrightarrow \\
\mathcal{X} \sqcap \mathcal{Y} \not\asymp \mathcal{A} &\Leftrightarrow \\
\mathcal{X} \left[\operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}}\right] \mathcal{Y}&
\end{aligned}$$

(used properties of generalized filter bases). □

Proposition 806.

1°. $(\mathsf{FCD})f$ is a monovalued funcoid if f is a monovalued reloid.
2°. $(\mathsf{FCD})f$ is an injective funcoid if f is an injective reloid.

Proof. We will prove only the first as the second is dual. Let f be a monovalued reloid. Then $f \circ f^{-1} \sqsubseteq \operatorname{id}^{\mathsf{RLD}(\operatorname{Dst} f)}$; $(\mathsf{FCD})(f \circ f^{-1}) \sqsubseteq \operatorname{id}^{\mathsf{FCD}(\operatorname{Dst} f)}$; $(\mathsf{FCD})f \circ ((\mathsf{FCD})f)^{-1} \sqsubseteq \operatorname{id}^{\mathsf{FCD}(\operatorname{Dst} f)}$ that is $(\mathsf{FCD})f$ is a monovalued funcoid. □

Proposition 807. $(\mathsf{FCD})(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) = \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ for every filters $\mathcal{A}$, $\mathcal{B}$.

Proof. $\mathcal{X} \left[(\mathsf{FCD})(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})\right] \mathcal{Y} \Leftrightarrow \forall F \in \operatorname{xyGR}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) : \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}$ (for every $\mathcal{X} \in \mathfrak{F}(\operatorname{Base}(\mathcal{A}))$, $\mathcal{Y} \in \mathfrak{F}(\operatorname{Base}(\mathcal{B}))$).

Evidently

$$\forall F \in \operatorname{xyGR}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) : \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y} \Rightarrow \forall A \in \mathcal{A}, B \in \mathcal{B} : \mathcal{X} \left[\uparrow^{\mathsf{FCD}(\operatorname{Base}(\mathcal{A}); \operatorname{Base}(\mathcal{B}))} (A \times B)\right] \mathcal{Y}.$$

Let $\forall A \in \mathcal{A}, B \in \mathcal{B} : \mathcal{X} \left[\uparrow^{\mathsf{FCD}(\operatorname{Base}(\mathcal{A}); \operatorname{Base}(\mathcal{B}))} (A \times B)\right] \mathcal{Y}$. Then if $F \in \operatorname{xyGR}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})$, there are $A \in \mathcal{A}$, $B \in \mathcal{B}$ such that $F \supseteq A \times B$. So $\mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}$. We have proved

$$\forall F \in \operatorname{xyGR}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) : \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y} \Leftrightarrow \forall A \in \mathcal{A}, B \in \mathcal{B} : \mathcal{X} \left[\uparrow^{\mathsf{FCD}(\operatorname{Base}(\mathcal{A}); \operatorname{Base}(\mathcal{B}))} (A \times B)\right] \mathcal{Y}.$$