

PROOF.

$$\begin{aligned} \langle (\text{FCD})(g \circ f) \rangle^* X &= \sqcap \left\{ \frac{\uparrow^{\text{Dst } g} \langle H \rangle^* X}{H \in \text{GR}(g \circ f)} \right\} = \\ &= \sqcap \left\{ \frac{\uparrow^{\text{Dst } g} \langle H \rangle^* X}{H \in \text{GR} \sqcap \left\{ \frac{\uparrow^{\text{RLD}(G \circ F)}}{F \in \text{xyGR } f, G \in \text{xyGR } g} \right\}} \right\}. \end{aligned}$$

Obviously

$$\begin{aligned} \sqcap \left\{ \frac{\uparrow^{\text{RLD}(G \circ F)}}{F \in \text{xyGR } f, G \in \text{xyGR } g} \right\} &= \\ \sqcap \langle \uparrow^{\text{RLD}} \rangle^* \text{xyGR} \sqcap \left\{ \frac{\uparrow^{\text{RLD}(G \circ F)}}{F \in \text{xyGR } f, G \in \text{xyGR } g} \right\}; \end{aligned}$$

from this by lemma 802 (taking into account that

$$\left\{ \frac{(G \circ F)}{F \in \text{xyGR } f, G \in \text{xyGR } g} \right\}$$

and

$$\text{xyGR} \sqcap \left\{ \frac{\uparrow^{\text{RLD}(G \circ F)}}{F \in \text{xyGR } f, G \in \text{xyGR } g} \right\}$$

are filter bases)

$$\sqcap \left\{ \frac{\uparrow^{\text{Dst } g} \langle H \rangle^* X}{H \in \text{GR} \sqcap \left\{ \frac{\uparrow^{\text{RLD}(G \circ F)}}{F \in \text{xyGR } f, G \in \text{xyGR } g} \right\}} \right\} = \left\{ \frac{\uparrow^{\text{Dst } g} \langle G \circ F \rangle^* X}{F \in \text{GR } f, G \in \text{GR } g} \right\}.$$

On the other side

$$\begin{aligned} \langle (\text{FCD})g \rangle \circ \langle (\text{FCD})f \rangle^* X &= \langle (\text{FCD})g \rangle \langle (\text{FCD})f \rangle^* X = \\ &= \langle (\text{FCD})g \rangle \sqcap \left\{ \frac{\uparrow^{\text{Dst } g} \langle F \rangle^* X}{F \in \text{xyGR } f} \right\} = \\ &= \sqcap \left\{ \frac{\langle \uparrow^{\text{FCD}} G \rangle \sqcap \left\{ \frac{\uparrow^{\text{Dst } g} \langle F \rangle^* X}{F \in \text{xyGR } f} \right\}}{G \in \text{xyGR } g} \right\}. \end{aligned}$$

Let's prove that  $\left\{ \frac{\langle F \rangle^* X}{F \in \text{xyGR } f} \right\}$  is a filter base. If  $A, B \in \left\{ \frac{\langle F \rangle^* X}{F \in \text{xyGR } f} \right\}$  then  $A = \langle F_1 \rangle^* X$ ,  $B = \langle F_2 \rangle^* X$  where  $F_1, F_2 \in \text{xyGR } f$ .  $A \cap B \supseteq \langle F_1 \sqcap F_2 \rangle^* X \in \left\{ \frac{\langle F \rangle^* X}{F \in \text{xyGR } f} \right\}$ . So  $\left\{ \frac{\langle F \rangle^* X}{F \in \text{xyGR } f} \right\}$  is really a filter base.

By theorem 599 we have

$$\langle \uparrow^{\text{FCD}} G \rangle \sqcap \left\{ \frac{\uparrow^{\text{Dst } g} \langle F \rangle^* X}{F \in \text{xyGR } f} \right\} = \sqcap \left\{ \frac{\uparrow^{\text{Dst } g} \langle G \rangle^* \langle F \rangle^* X}{F \in \text{xyGR } f} \right\}.$$