CHAPTER 8

Relationships between funcoids and reloids

8.1. Funcoid induced by a reloid

Every reloid f induces a funcoid $(\mathsf{FCD})f \in \mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)$ by the following formulas (for every $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{Y} \in \mathfrak{F}(\operatorname{Dst} f)$):

$$\mathcal{X}\ [(\mathsf{FCD})f]\ \mathcal{Y} \Leftrightarrow \forall F \in \operatorname{xyGR} f : \mathcal{X}\ \left[\uparrow^{\mathsf{FCD}} F\right]\ \mathcal{Y};$$

$$\langle(\mathsf{FCD})f\rangle\mathcal{X} = \bigsqcap\left\{\frac{\left\langle\uparrow^{\mathsf{FCD}} F\right\rangle\mathcal{X}}{F \in \operatorname{xyGR} f}\right\}.$$

We should prove that $(\mathsf{FCD})f$ is really a funcoid.

Proof. We need to prove that

$$\mathcal{X}\ [(\mathsf{FCD})f]\ \mathcal{Y} \Leftrightarrow \mathcal{Y} \sqcap \langle(\mathsf{FCD})f\rangle\mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \mathcal{X} \sqcap \left\langle(\mathsf{FCD})f^{-1}\right\rangle\mathcal{Y} \neq \bot^{\mathfrak{F}(\operatorname{Src} f)}.$$

The above formula is equivalent to:

$$\forall F \in \operatorname{xyGR} f : \mathcal{X}\ \left[\uparrow^{\mathsf{FCD}} F\right]\ \mathcal{Y} \Leftrightarrow$$
$$\mathcal{Y} \sqcap \bigsqcap\left\{\frac{\left\langle\uparrow^{\mathsf{FCD}} F\right\rangle\mathcal{X}}{F \in \operatorname{xyGR} f}\right\} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow$$
$$\mathcal{X} \sqcap \bigsqcap\left\{\frac{\left\langle\uparrow^{\mathsf{FCD}} F^{-1}\right\rangle\mathcal{Y}}{F \in \operatorname{xyGR} f}\right\} \neq \bot^{\mathfrak{F}(\operatorname{Src} f)}.$$

We have $\mathcal{Y} \sqcap \bigsqcap\left\{\frac{\left\langle\uparrow^{\mathsf{FCD}} F\right\rangle\mathcal{X}}{F \in \operatorname{xyGR} f}\right\} = \bigsqcap\left\{\frac{\mathcal{Y} \sqcap \left\langle\uparrow^{\mathsf{FCD}} F\right\rangle\mathcal{X}}{F \in \operatorname{xyGR} f}\right\}$.

Let's denote $W = \left\{\frac{\mathcal{Y} \sqcap \left\langle\uparrow^{\mathsf{FCD}} F\right\rangle\mathcal{X}}{F \in \operatorname{xyGR} f}\right\}$.

$\forall F \in \operatorname{xyGR} f : \mathcal{X}\ \left[\uparrow^{\mathsf{FCD}} F\right]\ \mathcal{Y} \Leftrightarrow \forall F \in \operatorname{xyGR} f : \mathcal{Y} \sqcap \left\langle\uparrow^{\mathsf{FCD}} F\right\rangle\mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \bot^{\mathfrak{F}(\operatorname{Dst} f)} \notin W$.

We need to prove only that $\bot^{\mathfrak{F}(\operatorname{Dst} f)} \notin W \Leftrightarrow \bigsqcap W \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}$. (The rest follows from symmetry.)

Let's prove that W is a generalized filter base. For this it's enough to prove that $V = \left\{\frac{\left\langle\uparrow^{\mathsf{FCD}} F\right\rangle\mathcal{X}}{F \in \operatorname{xyGR} f}\right\}$ is a generalized filter base. Let $\mathcal{A}, \mathcal{B} \in V$ that is $\mathcal{A} = \left\langle\uparrow^{\mathsf{FCD}} P\right\rangle\mathcal{X}$, $\mathcal{B} = \left\langle\uparrow^{\mathsf{FCD}} Q\right\rangle\mathcal{X}$ where $P, Q \in \operatorname{xyGR} f$. Then for $\mathcal{C} = \left\langle\uparrow^{\mathsf{FCD}} (P \sqcap Q)\right\rangle\mathcal{X}$ is true both $\mathcal{C} \in V$ and $\mathcal{C} \sqsubseteq \mathcal{A}, \mathcal{B}$. So V is a generalized filter base and thus W is a generalized filter base. □

Proposition 799. $(\mathsf{FCD}) \uparrow^{\mathsf{RLD}} f = \uparrow^{\mathsf{FCD}} f$ for every **Rel**-morphism f.

Proof. $\mathcal{X}\ \left[(\mathsf{FCD}) \uparrow^{\mathsf{RLD}} f\right]\ \mathcal{Y} \Leftrightarrow \forall F \in \operatorname{xyGR} \uparrow^{\mathsf{RLD}} f : \mathcal{X}\ \left[\uparrow^{\mathsf{FCD}} F\right]\ \mathcal{Y} \Leftrightarrow \mathcal{X}\ \left[\uparrow^{\mathsf{FCD}} f\right]\ \mathcal{Y}$ (for every $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{Y} \in \mathfrak{F}(\operatorname{Dst} f)$). □

Theorem 800. $\mathcal{X}\ [(\mathsf{FCD})f]\ \mathcal{Y} \Leftrightarrow \mathcal{X} \times^{\mathsf{RLD}} \mathcal{Y} \not\asymp f$ for every reloid f and $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{Y} \in \mathfrak{F}(\operatorname{Dst} f)$.