PROOF. We will prove only the first as the second is similar. Let

$$f = \bigsqcup\left\{\frac{\uparrow^{\mathrm{Src}\, f} \{\alpha\} \times^{\mathsf{RLD}} F(\alpha)}{\alpha \in \mathrm{Src}\, f}\right\} = \bigsqcup\left\{\frac{\uparrow^{\mathrm{Src}\, f} \{\alpha\} \times^{\mathsf{RLD}} G(\alpha)}{\alpha \in \mathrm{Src}\, f}\right\}$$

for some $F, G \in \mathfrak{F}(\mathrm{Dst}\, f)^{\mathrm{Src}\, f}$. We need to prove $F = G$. Let $\beta \in \mathrm{Src}\, f$.

$$f \sqcap (\uparrow^{\mathrm{Src}\, f} \{\beta\} \times^{\mathsf{RLD}} \top^{\mathfrak{F}(\mathrm{Dst}\, f)}) = \text{ (proposition 461)}$$

$$\bigsqcup\left\{\frac{(\uparrow^{\mathrm{Src}\, f} \{\alpha\} \times^{\mathsf{RLD}} F(\alpha)) \sqcap (\uparrow^{\mathrm{Src}\, f} \{\beta\} \times^{\mathsf{RLD}} \top^{\mathfrak{F}(\mathrm{Dst}\, f)})}{\alpha \in \mathrm{Src}\, f}\right\} =$$

$$\uparrow^{\mathrm{Src}\, f} \{\beta\} \times^{\mathsf{RLD}} F(\beta).$$

Similarly $f \sqcap (\uparrow^{\mathrm{Src}\, f} \{\beta\} \times^{\mathsf{RLD}} \top^{\mathfrak{F}(\mathrm{Dst}\, f)}) = \uparrow^{\mathrm{Src}\, f} \{\beta\} \times^{\mathsf{RLD}} G(\beta)$. Thus $\uparrow^{\mathrm{Src}\, f} \{\beta\} \times^{\mathsf{RLD}} F(\beta) = \uparrow^{\mathrm{Src}\, f} \{\beta\} \times^{\mathsf{RLD}} G(\beta)$ and so $F(\beta) = G(\beta)$. □

DEFINITION 785. *Completion* and *co-completion* of a reloid $f \in \mathsf{RLD}(A; B)$ are defined by the formulas:

$$\mathrm{Compl}\, f = \mathrm{Cor}^{(\mathsf{RLD}(A;B);\mathsf{ComplRLD}(A;B))} f; \quad \mathrm{CoCompl}\, f = \mathrm{Cor}^{(\mathsf{RLD}(A;B);\mathsf{CoComplRLD}(A;B))} f.$$

THEOREM 786. Atoms of the lattice $\mathsf{ComplRLD}(A; B)$ are exactly reloidal products of the form $\uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$ where $\alpha \in A$ and b is an ultrafilter on B.

PROOF. First, it's easy to see that $\uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$ are elements of $\mathsf{ComplRLD}(A; B)$. Also $\bot^{\mathsf{RLD}(A;B)}$ is an element of $\mathsf{ComplRLD}(A; B)$.

$\uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$ are atoms of $\mathsf{ComplRLD}(A; B)$ because they are atoms of $\mathsf{RLD}(A; B)$.

It remains to prove that if f is an atom of $\mathsf{ComplRLD}(A; B)$ then $f = \uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$ for some $\alpha \in A$ and an ultrafilter b on B.

Suppose f is a non-empty complete reloid. Then $\uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b \sqsubseteq f$ for some $\alpha \in A$ and an ultrafilter b on B. If f is an atom then $f = \uparrow^A \{\alpha\} \times^{\mathsf{RLD}} b$. □

OBVIOUS 787. $\mathsf{ComplRLD}(A; B)$ is an atomistic lattice.

PROPOSITION 788. $\mathrm{Compl}\, f = \bigsqcup\left\{\frac{f|_{\uparrow^{\mathrm{Src}\, f}\{\alpha\}}}{\alpha \in \mathrm{Src}\, f}\right\}$ for every reloid f.

PROOF. Let's denote R the right part of the equality to be proven.

That R is a complete reloid follows from the equality

$$f|_{\uparrow^{\mathrm{Src}\, f}\{\alpha\}} = \uparrow^{\mathrm{Src}\, f} \{\alpha\} \times^{\mathsf{RLD}} \mathrm{im}(f|_{\uparrow^{\mathrm{Src}\, f}\{\alpha\}}).$$

The only thing left to prove is that $g \sqsubseteq R$ for every complete reloid g such that $g \sqsubseteq f$.

Really let g be a complete reloid such that $g \sqsubseteq f$. Then

$$g = \bigsqcup\left\{\frac{\uparrow^{\mathrm{Src}\, f} \{\alpha\} \times^{\mathsf{RLD}} G(\alpha)}{\alpha \in \mathrm{Src}\, f}\right\}$$

for some function $G : \mathrm{Src}\, f \to \mathfrak{F}(\mathrm{Dst}\, f)$.

We have $\uparrow^{\mathrm{Src}\, f} \{\alpha\} \times^{\mathsf{RLD}} G(\alpha) = g|_{\uparrow^{\mathrm{Src}\, f}\{\alpha\}} \sqsubseteq f|_{\uparrow^{\mathrm{Src}\, f}\{\alpha\}}$. Thus $g \sqsubseteq R$. □

CONJECTURE 789. $\mathrm{Compl}\, f \sqcap \mathrm{Compl}\, g = \mathrm{Compl}(f \sqcap g)$ for every $f, g \in \mathsf{RLD}(A; B)$.

THEOREM 790. $\mathrm{Compl} \bigsqcup R = \bigsqcup \langle \mathrm{Compl} \rangle^* R$ for every set $R \in \mathscr{P}\mathsf{RLD}(A; B)$ for every sets A, B.