OBVIOUS 779. Principal reloids are complete and co-complete.

OBVIOUS 780. Join (on the lattice of reloids) of complete reloids is complete.

COROLLARY 781. $\mathsf{ComplRLD}$ (with the induced order) is a complete lattice.

THEOREM 782. A reloid which is both complete and co-complete is principal.

PROOF. Let f be a complete and co-complete reloid. We have

$$f = \bigsqcup\left\{\frac{\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{RLD}} G(\alpha)}{\alpha \in \operatorname{Src} f}\right\} \quad \text{and} \quad f = \bigsqcup\left\{\frac{H(\beta)\times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f}\{\beta\}}{\beta \in \operatorname{Dst} f}\right\}$$

for some functions $G : \operatorname{Src} f \to \mathfrak{F}(\operatorname{Dst} f)$ and $H : \operatorname{Dst} f \to \mathfrak{F}(\operatorname{Src} f)$. For every $\alpha \in \operatorname{Src} f$ we have

$$\begin{aligned}
&G(\alpha) = \\
&\operatorname{im} f|_{\uparrow^{\operatorname{Src} f}\{\alpha\}} = \\
&\operatorname{im}(f \sqcap (\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{RLD}} \top^{\mathfrak{F}(\operatorname{Dst} f)})) = \quad (*) \\
&\operatorname{im}\bigsqcup\left\{\frac{(H(\beta)\times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f}\{\beta\}) \sqcap (\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{RLD}} \top^{\mathfrak{F}(\operatorname{Dst} f)})}{\beta \in \operatorname{Dst} f}\right\} = \\
&\operatorname{im}\bigsqcup\left\{\frac{(H(\beta)\sqcap \uparrow^{\operatorname{Src} f}\{\alpha\})\times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f}\{\beta\}}{\beta \in \operatorname{Dst} f}\right\} = \\
&\operatorname{im}\bigsqcup\left\{\frac{\left(\begin{cases}\uparrow^{\operatorname{Src} f}\{\alpha\}\times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f}\{\beta\} & \text{if } H(\beta) \not\asymp \uparrow^{\operatorname{Src} f}\{\alpha\} \\ \bot^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} f)} & \text{if } H(\beta) \asymp \uparrow^{\operatorname{Src} f}\{\alpha\}\end{cases}\right)}{\beta \in \operatorname{Dst} f}\right\} = \\
&\operatorname{im}\bigsqcup\left\{\frac{\uparrow^{\operatorname{Src} f}\{\alpha\}\times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f}\{\beta\}}{\beta \in \operatorname{Dst} f, H(\beta) \not\asymp \uparrow^{\operatorname{Src} f}\{\alpha\}}\right\} = \\
&\operatorname{im}\bigsqcup\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} f)}\{(\alpha;\beta)\}}{\beta \in \operatorname{Dst} f, H(\beta) \not\asymp \uparrow^{\operatorname{Src} f}\{\alpha\}}\right\} = \\
&\bigsqcup\left\{\frac{\uparrow^{\operatorname{Dst} f}\{\beta\}}{\beta \in \operatorname{Dst} f, H(\beta) \not\asymp \uparrow^{\operatorname{Src} f}\{\alpha\}}\right\}
\end{aligned}$$

* proposition (461) was used.

Thus $G(\alpha)$ is a principal filter that is $G(\alpha) = \uparrow^{\operatorname{Dst} f} g(\alpha)$ for some $g : \operatorname{Src} f \to \operatorname{Dst} f$; $\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{RLD}} G(\alpha) = \uparrow^{\mathsf{RLD}(\operatorname{Src} f;\operatorname{Dst} f)} (\{\alpha\} \times g(\alpha))$; f is principal as a join of principal reloids. □

CONJECTURE 783. Composition of complete reloids is complete. FiXme: Solved.

THEOREM 784.

1°. For a complete reloid f there exists exactly one function $F \in \mathfrak{F}(\operatorname{Dst} f)^{\operatorname{Src} f}$ such that
$$f = \bigsqcup\left\{\frac{\uparrow^{\operatorname{Src} f}\{\alpha\} \times^{\mathsf{RLD}} F(\alpha)}{\alpha \in \operatorname{Src} f}\right\}.$$

2°. For a co-complete reloid f there exists exactly one function $F \in \mathfrak{F}(\operatorname{Src} f)^{\operatorname{Dst} f}$ such that
$$f = \bigsqcup\left\{\frac{F(\alpha)\times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f}\{\alpha\}}{\alpha \in \operatorname{Dst} f}\right\}.$$