

COROLLARY 769. Image and domain of reloids preserve joins.

PROOF. By properties of Galois connections and duality. \square

7.5. Categories of reloids

I will define two categories, the *category of reloids* and the *category of reloid triples*.

The *category of reloids* is defined as follows:

- Objects are small sets.
- The set of morphisms from a set A to a set B is $\text{RLD}(A; B)$.
- The composition is the composition of reloids.
- Identity morphism for a set is the identity reloid for that set.

To show it is really a category is trivial.

The *category of reloid triples* is defined as follows:

- Objects are small sets.
- The morphisms from a filter \mathcal{A} to a filter \mathcal{B} are triples $(\mathcal{A}; \mathcal{B}; f)$ where $f \in \text{RLD}(\text{Base}(\mathcal{A}); \text{Base}(\mathcal{B}))$ and $\text{dom } f \sqsubseteq \mathcal{A}$, $\text{im } f \sqsubseteq \mathcal{B}$.
- The composition is defined by the formula $(\mathcal{B}; \mathcal{C}; g) \circ (\mathcal{A}; \mathcal{B}; f) = (\mathcal{A}; \mathcal{C}; g \circ f)$.
- Identity morphism for a filter \mathcal{A} is $\text{id}_{\mathcal{A}}^{\text{RLD}}$.

To prove that it is really a category is trivial.

7.6. Monovalued and injective reloids

Following the idea of definition of monovalued morphism let's call *monovalued* such a reloid f that $f \circ f^{-1} \sqsubseteq \text{id}_{\text{im } f}^{\text{RLD}}$.

Similarly, I will call a reloid *injective* when $f^{-1} \circ f \sqsubseteq \text{id}_{\text{dom } f}^{\text{RLD}}$.

OBVIOUS 770. A reloid f is

- monovalued iff $f \circ f^{-1} \sqsubseteq \text{id}^{\text{RLD}(\text{Dst } f)}$;
- injective iff $f^{-1} \circ f \sqsubseteq \text{id}^{\text{RLD}(\text{Src } f)}$.

In other words, a reloid is monovalued (injective) when it is a monovalued (injective) morphism of the category of reloids.

Monovaluedness is dual of injectivity.

OBVIOUS 771.

- 1°. A morphism $(\mathcal{A}; \mathcal{B}; f)$ of the category of reloid triples is monovalued iff the reloid f is monovalued.
- 2°. A morphism $(\mathcal{A}; \mathcal{B}; f)$ of the category of reloid triples is injective iff the reloid f is injective.

THEOREM 772.

- 1°. A reloid f is a monovalued iff there exists a function (monovalued binary relation) $F \in \text{GR } f$.
- 2°. A reloid f is a injective iff there exists an injective binary relation $F \in \text{GR } f$.
- 3°. A reloid f is a both monovalued and injective iff there exists an injection (a monovalued and injective binary relation = injective function) $F \in \text{GR } f$.

PROOF. The reverse implications are obvious. Let's prove the direct implications: