

PROOF.

$$\begin{aligned}
& f \sqcap (\mathcal{A} \times^{\text{RLD}} \mathcal{B}) = \\
& f \sqcap (\mathcal{A} \times^{\text{RLD}} \top^{\mathfrak{F}}(\text{Dst } f)) \sqcap (\top^{\mathfrak{F}}(\text{Src } f) \times^{\text{RLD}} \mathcal{B}) = \\
& f|_{\mathcal{A}} \sqcap (\top^{\mathfrak{F}}(\text{Src } f) \times^{\text{RLD}} \mathcal{B}) = \\
& (f \circ \text{id}_{\mathcal{A}}^{\text{RLD}}) \sqcap (\top^{\mathfrak{F}}(\text{Src } f) \times^{\text{RLD}} \mathcal{B}) = \\
& ((f \circ \text{id}_{\mathcal{A}}^{\text{RLD}})^{-1} \sqcap (\top^{\mathfrak{F}}(\text{Src } f) \times^{\text{RLD}} \mathcal{B})^{-1})^{-1} = \\
& ((\text{id}_{\mathcal{A}}^{\text{RLD}} \circ f^{-1}) \sqcap (\mathcal{B} \times^{\text{RLD}} \top^{\mathfrak{F}}(\text{Src } f)))^{-1} = \\
& (\text{id}_{\mathcal{A}}^{\text{RLD}} \circ f \circ \text{id}_{\mathcal{B}}^{\text{RLD}})^{-1} = \\
& \text{id}_{\mathcal{B}}^{\text{RLD}} \circ f \circ \text{id}_{\mathcal{A}}^{\text{RLD}}.
\end{aligned}$$

□

THEOREM 767. $f|_{\uparrow^{\text{Src } f} \{\alpha\}} = \uparrow^{\text{Src } f} \{\alpha\} \times^{\text{RLD}} \text{im}(f|_{\uparrow^{\text{Src } f} \{\alpha\}})$ for every reloid f and $\alpha \in \text{Src } f$.

PROOF. First,

$$\begin{aligned}
& \text{im}(f|_{\uparrow^{\text{Src } f} \{\alpha\}}) = \\
& \sqcap \langle \uparrow^{\text{Dst } f} \rangle^* \langle \text{im} \rangle^* \text{GR}(f|_{\uparrow^{\text{Src } f} \{\alpha\}}) = \\
& \sqcap \langle \uparrow^{\text{Dst } f} \rangle^* \langle \text{im} \rangle^* \text{GR}(f \sqcap (\uparrow^{\text{Src } f} \{\alpha\} \times \top^{\mathfrak{F}}(\text{Dst } f))) = \\
& \sqcap \left\{ \frac{\uparrow^{\text{Dst } f} \text{im}(F \cap (\{\alpha\} \times \text{Dst } f))}{F \in \text{GR } f} \right\} = \\
& \sqcap \left\{ \frac{\uparrow^{\text{Dst } f} \text{im}(F|_{\{\alpha\}})}{F \in \text{GR } f} \right\}.
\end{aligned}$$

Taking this into account we have:

$$\begin{aligned}
& \uparrow^{\text{Src } f} \{\alpha\} \times^{\text{RLD}} \text{im}(f|_{\uparrow^{\text{Src } f} \{\alpha\}}) = \\
& \sqcap \left\{ \frac{\uparrow^{\text{RLD}(\text{Src } f; \text{Dst } f)} (\{\alpha\} \times K)}{K \in \text{im}(f|_{\uparrow^{\text{Src } f} \{\alpha\}})} \right\} = \\
& \sqcap \left\{ \frac{\uparrow^{\text{RLD}(\text{Src } f; \text{Dst } f)} (\{\alpha\} \times \text{im}(F|_{\{\alpha\}}))}{F \in \text{GR } f} \right\} = \\
& \sqcap \left\{ \frac{\uparrow^{\text{RLD}(\text{Src } f; \text{Dst } f)} (F|_{\{\alpha\}})}{F \in \text{GR } f} \right\} = \\
& \sqcap \left\{ \frac{\uparrow^{\text{RLD}(\text{Src } f; \text{Dst } f)} (F \cap (\{\alpha\} \times \text{Dst } f))}{F \in \text{GR } f} \right\} = \\
& \sqcap \left\{ \frac{\uparrow^{\text{RLD}(\text{Src } f; \text{Dst } f)} F}{F \in \text{GR } f} \right\} \sqcap \uparrow^{\text{RLD}(\text{Src } f; \text{Dst } f)} (\{\alpha\} \times \text{Dst } f) = \\
& f \sqcap \uparrow^{\text{RLD}(\text{Src } f; \text{Dst } f)} (\{\alpha\} \times \text{Dst } f) = \\
& f|_{\uparrow^{\text{Src } f} \{\alpha\}}.
\end{aligned}$$

□

LEMMA 768. $\lambda \mathcal{B} \in \mathfrak{F}(B) : \top^{\mathfrak{F}} \times^{\text{RLD}} \mathcal{B}$ is an upper adjoint of $\lambda f \in \text{RLD}(A; B) : \text{im } f$ (for every sets A, B).

PROOF. We need to prove $\text{im } f \sqsubseteq \mathcal{B} \Leftrightarrow \top^{\mathfrak{F}} \times^{\text{RLD}} \mathcal{B}$ what is obvious. □