THEOREM 743. $(h \circ g) \circ f = h \circ (g \circ f)$ for every composable reloids f, g, h.

PROOF. For two nonempty collections A and B of sets I will denote

$$A \sim B \Leftrightarrow \forall K \in A \exists L \in B : L \subseteq K \wedge \forall K \in B \exists L \in A : L \subseteq K.$$

It is easy to see that $\sim$ is a transitive relation.

I will denote $B \circ A = \left\{ \frac{L \circ K}{K \in A, L \in B} \right\}$.

Let first prove that for every nonempty collections of relations A, B, C

$$A \sim B \Rightarrow A \circ C \sim B \circ C.$$

Suppose $A \sim B$ and $P \in A \circ C$ that is $K \in A$ and $M \in C$ such that $P = K \circ M$. $\exists K' \in B : K' \subseteq K$ because $A \sim B$. We have $P' = K' \circ M \in B \circ C$. Obviously $P' \subseteq P$. So for every $P \in A \circ C$ there exists $P' \in B \circ C$ such that $P' \subseteq P$; the vice versa is analogous. So $A \circ C \sim B \circ C$.

$\operatorname{GR}((h \circ g) \circ f) \sim \operatorname{GR}(h \circ g) \circ \operatorname{GR} f$, $\operatorname{GR}(h \circ g) \sim (\operatorname{GR} h) \circ (\operatorname{GR} g)$. By proven above $\operatorname{GR}((h \circ g) \circ f) \sim (\operatorname{GR} h) \circ (\operatorname{GR} g) \circ (\operatorname{GR} f)$.

Analogously $\operatorname{GR}(h \circ (g \circ f)) \sim (\operatorname{GR} h) \circ (\operatorname{GR} g) \circ (\operatorname{GR} f)$.

So $\operatorname{GR}(h \circ (g \circ f)) \sim \operatorname{GR}((h \circ g) \circ f)$ what is possible only if $\operatorname{GR}(h \circ (g \circ f)) = \operatorname{GR}((h \circ g) \circ f)$. Thus $(h \circ g) \circ f = h \circ (g \circ f)$. □

THEOREM 744. For every reloid f:

1°. $f \circ f = \bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}}(F \circ F)}{F \in \operatorname{xyGR} f} \right\}$ if $\operatorname{Src} f = \operatorname{Dst} f$;
2°. $f^{-1} \circ f = \bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}}(F^{-1} \circ F)}{F \in \operatorname{xyGR} f} \right\}$;
3°. $f \circ f^{-1} = \bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}}(F \circ F^{-1})}{F \in \operatorname{xyGR} f} \right\}$.

PROOF. I will prove only 1° and 2° because 3° is analogous to 2°.

1°. It's enough to show that $\forall F, G \in \operatorname{xyGR} f \exists H \in \operatorname{xyGR} f : H \circ H \sqsubseteq G \circ F$. To prove it take $H = F \sqcap G$.

2°. It's enough to show that $\forall F, G \in \operatorname{xyGR} f \exists H \in \operatorname{xyGR} f : H^{-1} \circ H \sqsubseteq G^{-1} \circ F$. To prove it take $H = F \sqcap G$. Then $H^{-1} \circ H = (F \sqcap G)^{-1} \circ (F \sqcap G) \sqsubseteq G^{-1} \circ F$. □

THEOREM 745. For every sets A, B, C if $g, h \in \mathsf{RLD}(A; B)$ then

1°. $f \circ (g \sqcup h) = f \circ g \sqcup f \circ h$ for every $f \in \mathsf{RLD}(B; C)$;
2°. $(g \sqcup h) \circ f = g \circ f \sqcup h \circ f$ for every $f \in \mathsf{RLD}(C; A)$.

PROOF. We'll prove only the first as the second is dual.

By the infinite distributivity law for filters we have

$$f \circ g \sqcup f \circ h =$$
$$\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} (F \circ G)}{F \in \operatorname{xyGR} f, G \in \operatorname{xyGR} g} \right\} \sqcup \bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} (F \circ H)}{F \in \operatorname{xyGR} f, H \in \operatorname{xyGR} h} \right\} =$$
$$\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} (F_1 \circ G) \sqcup \uparrow^{\mathsf{RLD}} (F_2 \circ H)}{F_1, F_2 \in \operatorname{xyGR} f, G \in \operatorname{xyGR} g, H \in \operatorname{xyGR} h} \right\} =$$
$$\bigsqcap \left\{ \frac{\uparrow^{\mathsf{RLD}} ((F_1 \circ G) \sqcup (F_2 \circ H))}{F_1, F_2 \in \operatorname{xyGR} f, G \in \operatorname{xyGR} g, H \in \operatorname{xyGR} h} \right\}.$$