

COROLLARY 718. Every injective funcoid is metainjective.

CONJECTURE 719. Every metamonovalued funcoid is monovalued.

6.15. T_0 -, T_1 -, T_2 -, and T_3 -separable funcoids

For funcoids it can be generalized T_0 -, T_1 -, T_2 -, and T_3 - separability. Worthwhile note that T_0 and T_2 separability is defined through T_1 separability.

DEFINITION 720. Let call T_1 -separable such endofuncoid f that for every $\alpha, \beta \in \text{Ob } f$ is true

$$\alpha \neq \beta \Rightarrow \neg(\{\alpha\} [f]^* \{\beta\}).$$

PROPOSITION 721. An endofuncoid f is T_1 -separable iff $\text{Cor } f \sqsubseteq \text{id}^{\text{FCD}(\text{Ob } f)}$.

PROOF.

$$\forall x, y \in \text{Ob } f : (\{x\} [f]^* \{y\} \Rightarrow x = y) \Leftrightarrow$$

$$\forall x, y \in \text{Ob } f : (\{x\} [\text{Cor } f]^* \{y\} \Rightarrow x = y) \Leftrightarrow$$

$$\text{Cor } f \sqsubseteq \text{id}^{\text{FCD}(\text{Ob } f)}.$$

□

DEFINITION 722. Let call T_0 -separable such funcoid $f \in \text{FCD}(A; A)$ that $f \sqcap f^{-1}$ is T_1 -separable.

DEFINITION 723. Let call T_2 -separable such funcoid f that $f^{-1} \circ f$ is T_1 -separable.

For symmetric transitive funcoids T_1 - and T_2 -separability are the same (see theorem 210).

OBVIOUS 724. A funcoid f is T_2 -separable iff $\alpha \neq \beta \Rightarrow \langle f \rangle^* \{\alpha\} \not\prec \langle f \rangle^* \{\beta\}$ for every $\alpha, \beta \in \text{Src } f$.

DEFINITION 725. *Regular funcoid* is an endofuncoid f such that $\langle f \rangle \langle f^{-1} \rangle C \asymp \langle f \rangle^* \{p\} \Leftarrow p \notin C$ for every $p \in \text{Ob } f$ and $C \in \mathcal{P} \text{Ob } f$.

OBVIOUS 726. Funcoid f is regular iff:

$$1^\circ. \langle f \circ f^{-1} \rangle^* C \asymp \langle f \rangle^* \{p\} \Leftarrow p \notin C;$$

$$2^\circ. \langle f^{-1} \circ f \circ f^{-1} \rangle^* C \asymp^{\text{Ob } f} \{p\} \Leftarrow p \notin C;$$

$$3^\circ. \langle f^{-1} \circ f \circ f^{-1} \rangle^* C \sqsubseteq^{\text{Ob } f} C;$$

$$4^\circ. f^{-1} \circ f \circ f^{-1} \sqsubseteq \text{id}^{\text{FCD}(\text{Ob } f)}.$$

DEFINITION 727. An endofuncoid is T_3 - iff it is both T_2 - and regular.

6.16. Filters closed regarding a funcoid

DEFINITION 728. Let's call *closed* regarding a funcoid $f \in \text{FCD}(A; A)$ such filter $\mathcal{A} \in \mathfrak{F}(\text{Src } f)$ that $\langle f \rangle \mathcal{A} \sqsubseteq \mathcal{A}$.

This is a generalization of closedness of a set regarding an unary operation.

PROPOSITION 729. If I and J are closed (regarding some funcoid f), S is a set of closed filters on $\text{Src } f$, then

$$1^\circ. \mathcal{I} \sqcup \mathcal{J} \text{ is a closed filter;}$$

$$2^\circ. \prod S \text{ is a closed filter.}$$

PROOF. Let denote the given funcoid as f . $\langle f \rangle (\mathcal{I} \sqcup \mathcal{J}) = \langle f \rangle \mathcal{I} \sqcup \langle f \rangle \mathcal{J} \sqsubseteq \mathcal{I} \sqcup \mathcal{J}$, $\langle f \rangle \prod S \sqsubseteq \prod \langle \langle f \rangle \rangle^* S \sqsubseteq \prod S$. Consequently the filters $\mathcal{I} \sqcup \mathcal{J}$ and $\prod S$ are closed. □