THEOREM 709. Let μ, ν, π be endofuncoids. Let f be a co-complete open funcoid from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$ and g is an open funcoid from $\operatorname{Ob}\nu$ to $\operatorname{Ob}\pi$. Then $g \circ f$ is an open funcoid from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\pi$.

PROOF. Let $\operatorname{Compl}(f \circ \mu) \sqsupseteq \operatorname{Compl}(\nu \circ f)$ and $\operatorname{Compl}(g \circ \nu) \sqsupseteq \operatorname{Compl}(\pi \circ g)$.

$$\begin{aligned}
\operatorname{Compl}(g \circ f \circ \mu) &\sqsupseteq \\
\operatorname{Compl}(g \circ \operatorname{Compl}(f \circ \mu)) &\sqsupseteq \\
\operatorname{Compl}(g \circ \operatorname{Compl}(\nu \circ f)) &= \\
\operatorname{Compl}(g \circ \operatorname{Compl}(\nu) \circ f) &= \\
\operatorname{Compl}(g \circ \operatorname{Compl}(\nu)) \circ f &= \\
\operatorname{Compl}(g \circ \nu) \circ f &\sqsupseteq \\
\operatorname{Compl}(\pi \circ g) \circ f &= \\
\operatorname{Compl}(\pi \circ g \circ f).&
\end{aligned}$$

□

OBVIOUS 710. A funcoid f is open iff $f \circ \mu \sqsupseteq \operatorname{Compl}(\nu \circ f)$.

COROLLARY 711. A co-complete funcoid f is open iff $f \circ \mu \sqsupseteq (\operatorname{Compl}\nu) \circ f$. Thus f is open iff it is a continuous morphism from μ to $\operatorname{Compl}\nu$ *with the reverse order of funcoids.* (See a definition of a continuous morphism below.)

6.14. Monovalued and injective funcoids

Following the idea of definition of monovalued morphism let's call *monovalued* such a funcoid f that $f \circ f^{-1} \sqsubseteq \mathrm{id}^{\mathsf{FCD}}_{\operatorname{im} f}$.

Similarly, I will call a funcoid injective when $f^{-1} \circ f \sqsubseteq \mathrm{id}^{\mathsf{FCD}}_{\operatorname{dom} f}$.

OBVIOUS 712. A funcoid f is:

1°. monovalued iff $f \circ f^{-1} \sqsubseteq \mathrm{id}^{\mathsf{FCD}(\operatorname{Dst} f)}$;
2°. injective iff $f^{-1} \circ f \sqsubseteq \mathrm{id}^{\mathsf{FCD}(\operatorname{Src} f)}$.

In other words, a funcoid is monovalued (injective) when it is a monovalued (injective) morphism of the category of funcoids. Monovaluedness is dual of injectivity.

OBVIOUS 713.

1°. A morphism $(\mathcal{A}; \mathcal{B}; f)$ of the category of funcoid triples is monovalued iff the funcoid f is monovalued.
2°. A morphism $(\mathcal{A}; \mathcal{B}; f)$ of the category of funcoid triples is injective iff the funcoid f is injective.

THEOREM 714. The following statements are equivalent for a funcoid f:

1°. f is monovalued.
2°. $\forall a \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)} : \langle f \rangle a \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst} f)} \cup \{\bot^{\mathfrak{F}(\operatorname{Dst} f)}\}$.
3°. $\forall \mathcal{I}, \mathcal{J} \in \mathfrak{F}(\operatorname{Dst} f) : \langle f^{-1} \rangle (\mathcal{I} \sqcap \mathcal{J}) = \langle f^{-1} \rangle \mathcal{I} \sqcap \langle f^{-1} \rangle \mathcal{J}$.
4°. $\forall I, J \in \mathscr{P}(\operatorname{Dst} f) : \langle f^{-1} \rangle^* (I \cap J) = \langle f^{-1} \rangle^* I \sqcap \langle f^{-1} \rangle^* J$.

PROOF.