PROPOSITION 639. $\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ is really a funcoid and

$$\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle \mathcal{X} = \begin{cases} \mathcal{B} & \text{if } \mathcal{X} \not\asymp \mathcal{A} \\ \bot^{\mathfrak{F}(\operatorname{Base}(\mathcal{B}))} & \text{if } \mathcal{X} \asymp \mathcal{A}. \end{cases}$$

PROOF. Obvious. □

OBVIOUS 640. $\uparrow^{\mathsf{FCD}(U;V)} (A \times B) = \uparrow^U A \times \uparrow^V B$ for sets $A \subseteq U$ and $B \subseteq V$.

PROPOSITION 641. $f \sqsubseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \Leftrightarrow \operatorname{dom} f \sqsubseteq \mathcal{A} \wedge \operatorname{im} f \sqsubseteq \mathcal{B}$ for every $f \in \mathsf{FCD}(A;B)$ and $\mathcal{A} \in \mathfrak{F}(A)$, $\mathcal{B} \in \mathfrak{F}(B)$.

PROOF. If $f \sqsubseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ then $\operatorname{dom} f \sqsubseteq \operatorname{dom}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \sqsubseteq \mathcal{A}$, $\operatorname{im} f \sqsubseteq \operatorname{im}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \sqsubseteq \mathcal{B}$. If $\operatorname{dom} f \sqsubseteq \mathcal{A} \wedge \operatorname{im} f \sqsubseteq \mathcal{B}$ then

$$\forall \mathcal{X} \in \mathfrak{F}(A), \mathcal{Y} \in \mathfrak{F}(B) : (\mathcal{X} \; [f] \; \mathcal{Y} \Rightarrow \mathcal{X} \sqcap \mathcal{A} \neq \bot^{\mathfrak{F}(A)} \wedge \mathcal{Y} \sqcap \mathcal{B} \neq \bot^{\mathfrak{F}(B)});$$

consequently $f \sqsubseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$. □

The following theorem gives a formula for calculating an important particular case of a meet on the lattice of funcoids:

THEOREM 642. $f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) = \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}$ for every funcoid f and $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{B} \in \mathfrak{F}(\operatorname{Dst} f)$.

PROOF. $h \stackrel{\text{def}}{=} \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}$. For every $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$

$$\langle h \rangle \mathcal{X} = \left\langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}} \right\rangle \langle f \rangle \left\langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle \mathcal{X} = \mathcal{B} \sqcap \langle f \rangle (\mathcal{A} \sqcap \mathcal{X}).$$

From this, as easy to show, $h \sqsubseteq f$ and $h \sqsubseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$. If $g \sqsubseteq f \wedge g \sqsubseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ for a $g \in \mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)$ then $\operatorname{dom} g \sqsubseteq \mathcal{A}$, $\operatorname{im} g \sqsubseteq \mathcal{B}$,

$$\langle g \rangle \mathcal{X} = \mathcal{B} \sqcap \langle g \rangle (\mathcal{A} \sqcap \mathcal{X}) \sqsubseteq \mathcal{B} \sqcap \langle f \rangle (\mathcal{A} \sqcap \mathcal{X}) = \left\langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}} \right\rangle \langle f \rangle \left\langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle \mathcal{X} = \langle h \rangle \mathcal{X},$$

$g \sqsubseteq h$. So $h = f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B})$. □

COROLLARY 643. $f|_{\mathcal{A}} = f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \top^{\mathfrak{F}(\operatorname{Dst} f)})$ for every funcoid f and $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$.

PROOF. $f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \top^{\mathfrak{F}(\operatorname{Dst} f)}) = \mathrm{id}^{\mathsf{FCD}}_{\top^{\mathfrak{F}(\operatorname{Dst} f)}} \circ f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} = f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} = f|_{\mathcal{A}}$. □

COROLLARY 644. $f \not\asymp \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \Leftrightarrow \mathcal{A} \; [f] \; \mathcal{B}$ for every funcoid f and $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$, $\mathcal{B} \in \mathfrak{F}(\operatorname{Dst} f)$.

PROOF.

$$\begin{aligned} f \not\asymp \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} &\Leftrightarrow \\ \left\langle f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \right\rangle^* (\operatorname{Src} f) \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow \\ \left\langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle^* (\operatorname{Src} f) \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow \\ \left\langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{B}} \right\rangle \langle f \rangle \left\langle \mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle^* \top^{\mathfrak{F}(\operatorname{Src} f)} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow \\ \mathcal{B} \sqcap \langle f \rangle (\mathcal{A} \sqcap \top^{\mathfrak{F}(\operatorname{Src} f)}) \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow \\ \mathcal{B} \sqcap \langle f \rangle \mathcal{A} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow \\ \mathcal{A} \; [f] \; \mathcal{B}. \end{aligned}$$

□

COROLLARY 645. Every filtrator of funcoids is star-separable.

PROOF. The set of funcoidal products of principal filters is a separation subset of the lattice of funcoids. □