similarly $X\ \delta'\ I \cup J \Leftrightarrow X\ \delta'\ I \vee X\ \delta'\ J$ for suitable I and J. Let's continue δ' till a funcoid f (by the theorem 595):

$$\mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \forall X \in \mathcal{X}, Y \in \mathcal{Y} : X\ \delta'\ Y.$$

The reverse of (10) implication is trivial, so

$$\forall X \in a, Y \in b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^A X : x\ \delta\ y \Leftrightarrow a\ \delta\ b.$$

Also

$$\begin{aligned} \forall X \in a, Y \in b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^A X : x\ \delta\ y \Leftrightarrow \\ \forall X \in a, Y \in B : X\ \delta'\ Y \Leftrightarrow \\ a\ [f]\ b. \end{aligned}$$

So $a\ \delta\ b \Leftrightarrow a\ [f]\ b$, that is $[f]$ is a continuation of δ.

□

One of uses of the previous theorem is the proof of the following theorem:

THEOREM 635. If A and B are sets, $R \in \mathscr{P}\mathsf{FCD}(A; B)$, $x \in \operatorname{atoms}^{\mathfrak{F}(A)}$, $y \in \operatorname{atoms}^{\mathfrak{F}(B)}$, then

1°. $\langle \bigsqcap R \rangle x = \bigsqcap \left\{ \frac{\langle f \rangle x}{f \in R} \right\}$;
2°. $x\ [\bigsqcap R]\ y \Leftrightarrow \forall f \in R : x\ [f]\ y$.

PROOF.
2°. Let denote $x\ \delta\ y \Leftrightarrow \forall f \in R : x\ [f]\ y$. For every $a \in \operatorname{atoms}^{\mathfrak{F}(A)}$, $b \in \operatorname{atoms}^{\mathfrak{F}(B)}$

$$\begin{aligned} \forall X \in a, Y \in b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^B Y : x\ \delta\ y \Rightarrow \\ \forall f \in R, X \in a, Y \in b \exists x \in \operatorname{atoms} \uparrow^A X, y \in \operatorname{atoms} \uparrow^B Y : x\ [f]\ y \Rightarrow \\ \forall f \in R, X \in a, Y \in b : X\ [f]^*\ Y \Rightarrow \\ \forall f \in R : a\ [f]\ b \Leftrightarrow \\ a\ \delta\ b. \end{aligned}$$

So by theorem (634), δ can be continued till $[p]$ for some funcoid $p \in \mathsf{FCD}(A; B)$. For every funcoid $q \in \mathsf{FCD}(A; B)$ such that $\forall f \in R : q \sqsubseteq f$ we have

$$x\ [q]\ y \Rightarrow \forall f \in R : x\ [f]\ y \Leftrightarrow x\ \delta\ y \Leftrightarrow x\ [p]\ y,$$

so $q \sqsubseteq p$. Consequently $p = \bigsqcap R$.
From this $x\ [\bigsqcap R]\ y \Leftrightarrow \forall f \in R : x\ [f]\ y$.
1°. From the former

$$\begin{aligned} y \in \operatorname{atoms}\left\langle \bigsqcap R \right\rangle x \Leftrightarrow \\ y \sqcap \left\langle \bigsqcap R \right\rangle x \neq \bot^{\mathfrak{F}(B)} \Leftrightarrow \\ \forall f \in R : y \sqcap \langle f \rangle x \neq \bot^{\mathfrak{F}(B)} \Leftrightarrow \\ y \in \bigsqcap \langle \operatorname{atoms} \rangle^* \left\{ \frac{\langle f \rangle x}{f \in R} \right\} \Leftrightarrow \\ y \in \operatorname{atoms} \bigsqcap \left\{ \frac{\langle f \rangle x}{f \in R} \right\} \end{aligned}$$

for every $y \in \operatorname{atoms}^{\mathfrak{F}(A)}$. From this it follows $\langle \bigsqcap R \rangle x = \bigsqcap \left\{ \frac{\langle f \rangle x}{f \in R} \right\}$.

□