To show it is really a category is trivial.

The *category of funcoid triples* is defined as follows:

- Objects are filters on small sets.
- The morphisms from a filter $\mathcal{A}$ to a filter $\mathcal{B}$ are triples $(\mathcal{A};\mathcal{B};f)$ where $f\in\mathsf{FCD}(\operatorname{Base}(\mathcal{A});\operatorname{Base}(\mathcal{B}))$ and $\operatorname{dom}f\sqsubseteq\mathcal{A}\wedge\operatorname{im}f\sqsubseteq\mathcal{B}$.
- The composition is defined by the formula $(\mathcal{B};\mathcal{C};g)\circ(\mathcal{A};\mathcal{B};f)=(\mathcal{A};\mathcal{C};g\circ f)$.
- Identity morphism for a filter $\mathcal{A}$ is $\operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}}$.

To prove that it is really a category is trivial.

6.8. Specifying funcoids by functions or relations on atomic filters

THEOREM 632. For every funcoid f and $\mathcal{X}\in\mathfrak{F}(\operatorname{Src}f)$, $\mathcal{Y}\in\mathfrak{F}(\operatorname{Dst}f)$

1°. $\langle f\rangle\mathcal{X}=\bigsqcup\langle\langle f\rangle\rangle^*\operatorname{atoms}\mathcal{X}$;

2°. $\mathcal{X}\ [f]\ \mathcal{Y}\Leftrightarrow\exists x\in\operatorname{atoms}\mathcal{X},y\in\operatorname{atoms}\mathcal{Y}:x\ [f]\ y$.

PROOF.

1°.

$$\mathcal{Y}\sqcap\langle f\rangle\mathcal{X}\neq\bot^{\mathfrak{F}(\operatorname{Dst}f)}\Leftrightarrow$$
$$\mathcal{X}\sqcap\left\langle f^{-1}\right\rangle\mathcal{Y}\neq\bot^{\mathfrak{F}(\operatorname{Src}f)}\Leftrightarrow$$
$$\exists x\in\operatorname{atoms}\mathcal{X}:x\sqcap\left\langle f^{-1}\right\rangle\mathcal{Y}\neq\bot^{\mathfrak{F}(\operatorname{Src}f)}\Leftrightarrow$$
$$\exists x\in\operatorname{atoms}\mathcal{X}:\mathcal{Y}\sqcap\langle f\rangle x\neq\bot^{\mathfrak{F}(\operatorname{Dst}f)}.$$

$\partial\langle f\rangle\mathcal{X}=\bigsqcup\langle\partial\rangle^*\langle\langle f\rangle\rangle^*\operatorname{atoms}\mathcal{X}=\partial\bigsqcup\langle\langle f\rangle\rangle^*\operatorname{atoms}\mathcal{X}$. So $\langle f\rangle\mathcal{X}=\bigsqcup\langle\langle f\rangle\rangle^*\operatorname{atoms}\mathcal{X}$ by proposition 469.

2°. If $\mathcal{X}\ [f]\ \mathcal{Y}$, then $\mathcal{Y}\sqcap\langle f\rangle\mathcal{X}\neq\bot^{\mathfrak{F}(\operatorname{Dst}f)}$, consequently there exists $y\in\operatorname{atoms}\mathcal{Y}$ such that $y\sqcap\langle f\rangle\mathcal{X}\neq\bot^{\mathfrak{F}(\operatorname{Dst}f)}$, $\mathcal{X}\ [f]\ y$. Repeating this second time we get that there exists $x\in\operatorname{atoms}\mathcal{X}$ such that $x\ [f]\ y$. From this it follows

$$\exists x\in\operatorname{atoms}\mathcal{X},y\in\operatorname{atoms}\mathcal{Y}:x\ [f]\ y.$$

The reverse is obvious.

□

COROLLARY 633. Let f be a funcoid.

- The value of f can be restored knowing $\langle f\rangle|_{\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src}f)}}$.
- The value of f can be restored knowing $[f]|_{\operatorname{atoms}^{\mathfrak{F}(\operatorname{Src}f)}\times\operatorname{atoms}^{\mathfrak{F}(\operatorname{Dst}f)}}$.

THEOREM 634. Let A and B be sets.

1°. A function $\alpha\in\mathfrak{F}(B)^{\operatorname{atoms}^{\mathfrak{F}(A)}}$ such that (for every $a\in\operatorname{atoms}^{\mathfrak{F}(A)}$)

$$\alpha a\sqsubseteq\bigsqcap\left\langle\bigsqcup\circ\langle\alpha\rangle^*\circ\operatorname{atoms}\circ\uparrow^A\right\rangle^*a\tag{8}$$

can be continued to the function $\langle f\rangle$ for a unique $f\in\mathsf{FCD}(A;B)$;

$$\langle f\rangle\mathcal{X}=\bigsqcup\langle\alpha\rangle^*\operatorname{atoms}\mathcal{X}\tag{9}$$

for every $\mathcal{X}\in\mathfrak{F}(A)$.

2°. A relation $\delta\in\mathscr{P}(\operatorname{atoms}^{\mathfrak{F}(A)}\times\operatorname{atoms}^{\mathfrak{F}(B)})$ such that (for every $a\in\operatorname{atoms}^{\mathfrak{F}(A)}$, $b\in\operatorname{atoms}^{\mathfrak{F}(B)}$)

$$\forall X\in a,Y\in b\exists x\in\operatorname{atoms}\uparrow^A X,y\in\operatorname{atoms}\uparrow^A X:x\ \delta\ y\Rightarrow a\ \delta\ b\tag{10}$$

can be continued to the relation $[f]$ for a unique $f\in\mathsf{FCD}(A;B)$;

$$\mathcal{X}\ [f]\ \mathcal{Y}\Leftrightarrow\exists x\in\operatorname{atoms}\mathcal{X},y\in\operatorname{atoms}\mathcal{X}:x\ \delta\ y\tag{11}$$

for every $\mathcal{X}\in\mathfrak{F}(A)$, $\mathcal{Y}\in\mathfrak{F}(B)$.