1°. $\operatorname{im} \uparrow^{\mathsf{FCD}(A;B)} f = \uparrow^B \operatorname{im} f$;
2°. $\operatorname{dom} \uparrow^{\mathsf{FCD}(A;B)} f = \uparrow^A \operatorname{dom} f$.

PROPOSITION 623. $\langle f\rangle\mathcal{X} = \langle f\rangle(\mathcal{X} \sqcap \operatorname{dom} f)$ for every funcoid f, $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$.

PROOF. For every $\mathcal{Y} \in \mathfrak{F}(\operatorname{Dst} f)$ we have

$$\begin{aligned}\mathcal{Y} \sqcap \langle f\rangle(\mathcal{X} \sqcap \operatorname{dom} f) \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow\\ \mathcal{X} \sqcap \operatorname{dom} f \sqcap \langle f^{-1}\rangle\mathcal{Y} \neq \bot^{\mathfrak{F}(\operatorname{Src} f)} &\Leftrightarrow\\ \mathcal{X} \sqcap \operatorname{im} f^{-1} \sqcap \langle f^{-1}\rangle\mathcal{Y} \neq \bot^{\mathfrak{F}(\operatorname{Src} f)} &\Leftrightarrow\\ \mathcal{X} \sqcap \langle f^{-1}\rangle\mathcal{Y} \neq \bot^{\mathfrak{F}(\operatorname{Src} f)} &\Leftrightarrow\\ \mathcal{Y} \sqcap \langle f\rangle\mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}&.\end{aligned}$$

Thus $\langle f\rangle(\mathcal{X} \sqcap \operatorname{dom} f) = \langle f\rangle\mathcal{X}$ because the lattice of filters is separable. □

PROPOSITION 624. $\langle f\rangle\mathcal{X} = \operatorname{im}(f|_{\mathcal{X}})$ for every funcoid f, $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$.

PROOF.

$$\begin{aligned}\operatorname{im}(f|_{\mathcal{X}}) &=\\ \left\langle f \circ \operatorname{id}^{\mathsf{FCD}}_{\mathcal{X}}\right\rangle\top^{\mathfrak{F}(\operatorname{Src} f)} &=\\ \langle f\rangle\left\langle \operatorname{id}^{\mathsf{FCD}}_{\mathcal{X}}\right\rangle\top^{\mathfrak{F}(\operatorname{Src} f)} &=\\ \langle f\rangle(\mathcal{X} \sqcap \top^{\mathfrak{F}(\operatorname{Src} f)}) &=\\ \langle f\rangle\mathcal{X}&.\end{aligned}$$

□

PROPOSITION 625. $\mathcal{X} \sqcap \operatorname{dom} f \neq \bot^{\mathfrak{F}(\operatorname{Src} f)} \Leftrightarrow \langle f\rangle\mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}$ for every funcoid f and $\mathcal{X} \in \mathfrak{F}(\operatorname{Src} f)$.

PROOF.

$$\begin{aligned}\mathcal{X} \sqcap \operatorname{dom} f \neq \bot^{\mathfrak{F}(\operatorname{Src} f)} &\Leftrightarrow\\ \mathcal{X} \sqcap \langle f^{-1}\rangle\top^{\mathfrak{F}(\operatorname{Dst} f)} \neq \bot^{\mathfrak{F}(\operatorname{Src} f)} &\Leftrightarrow\\ \top^{\mathfrak{F}(\operatorname{Dst} f)} \sqcap \langle f\rangle\mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow\\ \langle f\rangle\mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}&.\end{aligned}$$

□

COROLLARY 626. $\operatorname{dom} f = \bigsqcup\left\{ \frac{a \in \operatorname{atoms}^{\mathfrak{F}(\operatorname{Src} f)}}{\langle f\rangle a \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}} \right\}$.

PROOF. This follows from the fact that $\mathfrak{F}(\operatorname{Src} f)$ is an atomistic lattice. □

PROPOSITION 627. $\operatorname{dom}(f|_{\mathcal{A}}) = \mathcal{A} \sqcap \operatorname{dom} f$ for every funcoid f and $\mathcal{A} \in \mathfrak{F}(\operatorname{Src} f)$.

PROOF.

$$\begin{aligned}\operatorname{dom}(f|_{\mathcal{A}}) &=\\ \operatorname{im}(\operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}} \circ f^{-1}) &=\\ \left\langle \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}}\right\rangle\langle f^{-1}\rangle\top^{\mathfrak{F}(\operatorname{Dst} f)} &=\\ \mathcal{A} \sqcap \langle f^{-1}\rangle\top^{\mathfrak{F}(\operatorname{Dst} f)} &=\\ \mathcal{A} \sqcap \operatorname{dom} f&.\end{aligned}$$