Proof.

$$\bigsqcap\left\{\frac{\langle F\rangle\mathcal{X}}{F\in\operatorname{up}f}\right\}=$$
$$\bigsqcap\left\{\frac{\langle\langle F\rangle^*\rangle^*\mathcal{X}}{F\in\operatorname{up}f}\right\}=$$
$$\bigsqcap\left\{\frac{\bigsqcap\left\{\frac{\langle F\rangle^* X}{X\in\mathcal{X}}\right\}}{F\in\operatorname{up}f}\right\}=$$
$$\bigsqcap\left\{\frac{\bigsqcap\left\{\frac{\langle F\rangle^* X}{F\in\operatorname{up}f}\right\}}{X\in\mathcal{X}}\right\}=$$
$$\bigsqcap\left\{\frac{\langle f\rangle^* X}{X\in\mathcal{X}}\right\}=$$
$$\langle f\rangle\mathcal{X}$$

(the lemma used). □

Conjecture 603. Every filtrator of funcoids is:FiXme: Solved. See rewrite-plan.pdf

1°. with separable core;
2°. with co-separable core.

Below it is shown that $\mathsf{FCD}(A;B)$ are complete lattices for every sets A and B. We will apply lattice operations to subsets of such sets without explicitly mentioning $\mathsf{FCD}(A;B)$.

Theorem 604. $\mathsf{FCD}(A;B)$ is a complete lattice (for every sets A and B). For every $R\in\mathscr{P}\mathsf{FCD}(A;B)$ and $X\in\mathscr{P}A$, $Y\in\mathscr{P}B$

1°. $X\left[\bigsqcup R\right]^* Y\Leftrightarrow\exists f\in R: X\left[f\right]^* Y$;
2°. $\langle\bigsqcup R\rangle^* X=\bigsqcup\left\{\frac{\langle f\rangle^* X}{f\in R}\right\}$.

Proof. Accordingly [26] to prove that it is a complete lattice it's enough to prove existence of all joins.

2°. $\alpha X\overset{\text{def}}{=}\bigsqcup\left\{\frac{\langle f\rangle^* X}{f\in R}\right\}$. We have $\alpha\emptyset=\bot^{\mathfrak{F}(\operatorname{Dst}f)}$;

$$\alpha(I\cup J)=$$
$$\bigsqcup\left\{\frac{\langle f\rangle^*(I\cup J)}{f\in R}\right\}=$$
$$\bigsqcup\left\{\frac{\langle f\rangle^* I\sqcup\langle f\rangle^* J}{f\in R}\right\}=$$
$$\bigsqcup\left\{\frac{\langle f\rangle^* I}{f\in R}\right\}\sqcup\bigsqcup\left\{\frac{\langle f\rangle^* J}{f\in R}\right\}=$$
$$\alpha I\sqcup\alpha J.$$

So $\langle h\rangle^*=\alpha$ for some funcoid h. Obviously

$$\forall f\in R: h\sqsupseteq f. \tag{7}$$

And h is the least funcoid for which holds the condition (7). So $h=\bigsqcup R$.