1°. Let define the relation $\delta \in \mathscr{P}(\mathscr{P}A \times \mathscr{P}B)$ by the formula $X \; \delta \; Y \Leftrightarrow \uparrow^B \sqcap \alpha X \neq \bot^{\mathfrak{F}(B)}$.

That $\neg(I \; \delta \; \emptyset)$ and $\neg(\emptyset \; \delta \; I)$ is obvious. We have

$$\begin{aligned} I \cup J \; \delta \; K &\Leftrightarrow \\ \uparrow^B K \sqcap \alpha(I \cup J) \neq \bot^{\mathfrak{F}(B)} &\Leftrightarrow \\ \uparrow^B K \sqcap (\alpha I \sqcup \alpha J) \neq \bot^{\mathfrak{F}(B)} &\Leftrightarrow \\ \uparrow^B K \sqcap \alpha I \neq \bot^{\mathfrak{F}(B)} \vee \uparrow^B K \sqcap \alpha J \neq \bot^{\mathfrak{F}(B)} &\Leftrightarrow \\ I \; \delta \; K \vee J \; \delta \; K \end{aligned}$$

and

$$\begin{aligned} K \; \delta \; I \cup J &\Leftrightarrow \\ \uparrow^B (I \cup J) \sqcap \alpha K \neq \bot^{\mathfrak{F}(B)} &\Leftrightarrow \\ (\uparrow^B I \sqcup \uparrow^B J) \sqcap \alpha K \neq \bot^{\mathfrak{F}(B)} &\Leftrightarrow \\ \uparrow^B I \sqcap \alpha K \neq \bot^{\mathfrak{F}(B)} \vee \uparrow^B J \sqcap \alpha K \neq \bot^{\mathfrak{F}(B)} &\Leftrightarrow \\ K \; \delta \; I \vee K \; \delta \; J. \end{aligned}$$

That is the formulas (5) are true.

Accordingly to the above there exists a funcoid f such that

$$\mathcal{X} \; [f] \; \mathcal{Y} \Leftrightarrow \forall X \in \mathcal{X}, Y \in \mathcal{Y} : X \; \delta \; Y.$$

For every $X \in \mathscr{P}A$, $Y \in \mathscr{P}B$ we have:

$$\uparrow^B Y \sqcap \langle f \rangle \uparrow^A X \neq \bot^{\mathfrak{F}(B)} \Leftrightarrow \uparrow^A X \; [f] \uparrow^B Y \Leftrightarrow X \; \delta \; Y \Leftrightarrow \uparrow^B Y \sqcap \alpha X \neq \bot^{\mathfrak{F}(B)},$$

consequently $\forall X \in \mathscr{P}A : \alpha X = \langle f \rangle \uparrow^A X = \langle f \rangle^* X$.

□

Note that by the last theorem to every proximity δ corresponds a unique funcoid. So funcoids are a generalization of (quasi-)proximity structures. Reverse funcoids can be considered as a generalization of conjugate quasi-proximity.

DEFINITION 596. Any (multivalued) function $F : A \to B$ corresponds to a funcoid $\uparrow^{\mathsf{FCD}(A;B)} F \in \mathsf{FCD}(A;B)$, where by definition $\langle \uparrow^{\mathsf{FCD}(A;B)} F \rangle \mathcal{X} = \bigsqcap \langle \uparrow^B \rangle^* \langle \langle F \rangle^* \rangle^* \mathcal{X}$ for every $\mathcal{X} \in \mathfrak{F}(A)$.

Using the last theorem it is easy to show that this definition is monovalued and does not contradict to former stuff. (Take $\alpha = \uparrow^B \circ \langle F \rangle^*$.)

DEFINITION 597. $\uparrow^{\mathsf{FCD}} f = (\operatorname{Src} f; \operatorname{Dst} f; \uparrow^{\mathsf{FCD}(\operatorname{Src} f; \operatorname{Dst} f)} \operatorname{GR} f)$ for every **Rel**-morphism f.

DEFINITION 598. Funcoids corresponding to a binary relation (= multivalued function) are called *principal funcoids*.

We may equate principal funcoids with corresponding binary relations by the method of appendix B in [29]. This is useful for describing relationships of funcoids and binary relations, such as for the formulas of continuous functions and continuous funcoids (see below).

THEOREM 599. If S is a generalized filter base on $\operatorname{Src} f$ then $\langle f \rangle \bigsqcap S = \bigsqcap \langle \langle f \rangle \rangle^* S$ for every funcoid f.