Analogously $\mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \forall X \in \mathcal{X} : \uparrow^{\operatorname{Src} f} X\ [f]\ \mathcal{Y}$. Combining these two equivalences we get

$$\mathcal{X}\ [f]\ \mathcal{Y} \Leftrightarrow \forall X \in \mathcal{X}, Y \in \mathcal{Y} : \uparrow^{\operatorname{Src} f} X\ [f] \uparrow^{\operatorname{Dst} f} Y \Leftrightarrow \forall X \in \mathcal{X}, Y \in \mathcal{Y} : X\ [f]^*\ Y.$$

1°.

$$\begin{aligned} \mathcal{Y} \sqcap \langle f \rangle \mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} &\Leftrightarrow \\ \mathcal{X}\ [f]\ \mathcal{Y} &\Leftrightarrow \\ \forall X \in \mathcal{X} : \uparrow^{\operatorname{Src} f} X\ [f]\ \mathcal{Y} &\Leftrightarrow \\ \forall X \in \mathcal{X} : \mathcal{Y} \sqcap \langle f \rangle^* X \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}&. \end{aligned}$$

Let's denote $W = \left\{ \frac{\mathcal{Y} \sqcap \langle f \rangle^* X}{X \in \mathcal{X}} \right\}$. We will prove that W is a generalized filter base. To prove this it is enough to show that $V = \left\{ \frac{\langle f \rangle^* X}{X \in \mathcal{X}} \right\}$ is a generalized filter base.

Let $\mathcal{P}, \mathcal{Q} \in V$. Then $\mathcal{P} = \langle f \rangle^* A$, $\mathcal{Q} = \langle f \rangle^* B$ where $A, B \in \mathcal{X}$; $A \cap B \in \mathcal{X}$ and $\mathcal{R} \sqsubseteq \mathcal{P} \sqcap \mathcal{Q}$ for $\mathcal{R} = \langle f \rangle^* (A \cap B) \in V$. So V is a generalized filter base and thus W is a generalized filter base.

$\bot^{\mathfrak{F}(\operatorname{Dst} f)} \notin W \Leftrightarrow \bigsqcap W \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}$ by properties of generalized filter bases. That is

$$\forall X \in \mathcal{X} : \mathcal{Y} \sqcap \langle f \rangle^* X \not\asymp \bot^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \mathcal{Y} \sqcap \bigsqcap \langle \langle f \rangle^* \rangle^* \mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}.$$

Comparing with the above, $\mathcal{Y} \sqcap \langle f \rangle \mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)} \Leftrightarrow \mathcal{Y} \sqcap \bigsqcap \langle \langle f \rangle^* \rangle^* \mathcal{X} \neq \bot^{\mathfrak{F}(\operatorname{Dst} f)}$. So $\langle f \rangle \mathcal{X} = \bigsqcap \langle \langle f \rangle^* \rangle^* \mathcal{X}$ because the lattice of filters is separable.

□

Corollary 593. Let f be a funcoid.

1°. The value of f can be restored knowing $\langle f \rangle^*$.
2°. The value of f can be restored knowing $[f]^*$.

Proposition 594. For every $f \in \mathsf{FCD}(A; B)$ we have (for every $I, J \in \mathscr{P} A$)

$$\langle f \rangle^* \emptyset = \bot^{\mathfrak{F}(B)}, \quad \langle f \rangle^* (I \cup J) = \langle f \rangle^* I \sqcup \langle f \rangle^* J$$

and

$$\neg (I\ [f]^*\ \emptyset), \quad I \cup J\ [f]^*\ K \Leftrightarrow I\ [f]^*\ K \vee J\ [f]^*\ K \quad (\text{for every } I, J \in \mathscr{P} A,\ K \in \mathscr{P} B),$$
$$\neg (\emptyset\ [f]^*\ I), \quad K\ [f]^*\ I \cup J \Leftrightarrow K\ [f]^*\ I \vee K\ [f]^*\ J \quad (\text{for every } I, J \in \mathscr{P} B,\ K \in \mathscr{P} A).$$

Proof. $\langle f \rangle^* \emptyset = \langle f \rangle \uparrow^A \emptyset = \langle f \rangle \bot^{\mathfrak{F}(A)} = \bot^{\mathfrak{F}(B)}$;

$$\langle f \rangle^* (I \cup J) = \langle f \rangle \uparrow^A (I \cup J) = \langle f \rangle \uparrow^A I \sqcup \langle f \rangle \uparrow^A J = \langle f \rangle^* I \sqcup \langle f \rangle^* J.$$

$I\ [f]^*\ \emptyset \Leftrightarrow \bot^{\mathfrak{F}(B)} \not\asymp \langle f \rangle \uparrow^A I \Leftrightarrow 0$;

$$\begin{aligned} I \cup J\ [f]^*\ K &\Leftrightarrow \\ \uparrow^A (I \cup J)\ [f] \uparrow^B K &\Leftrightarrow \\ \uparrow^B K \not\asymp \langle f \rangle \uparrow^A (I \cup J) &\Leftrightarrow \\ \uparrow^B K \not\asymp \langle f \rangle^* (I \cup J) &\Leftrightarrow \\ \uparrow^B K \not\asymp \langle f \rangle^* I \sqcup \langle f \rangle^* J &\Leftrightarrow \\ \uparrow^B K \not\asymp \langle f \rangle^* I \vee \uparrow^B K \not\asymp \langle f \rangle^* J &\Leftrightarrow \\ I\ [f]^*\ K \vee J\ [f]^*\ K&. \end{aligned}$$

The rest follows from symmetry. □