PROOF. Let $f, g \in \mathsf{FCD}(A; B)$.

Obviously, $\langle f \rangle = \langle g \rangle \Rightarrow [f] = [g]$ and $\langle f^{-1} \rangle = \langle g^{-1} \rangle \Rightarrow [f] = [g]$. So it's enough to prove that $[f] = [g] \Rightarrow \langle f \rangle = \langle g \rangle$.

Provided that $[f] = [g]$ we have $\mathcal{Y} \not\asymp \langle f \rangle \mathcal{X} \Leftrightarrow \mathcal{X} \; [f] \; \mathcal{Y} \Leftrightarrow \mathcal{X} \; [g] \; \mathcal{Y} \Leftrightarrow \mathcal{Y} \not\asymp \langle g \rangle \mathcal{X}$ and consequently $\langle f \rangle \mathcal{X} = \langle g \rangle \mathcal{X}$ for every $\mathcal{X} \in \mathfrak{F}(A)$, $\mathcal{Y} \in \mathfrak{F}(B)$ because a set of filters is separable, thus $\langle f \rangle = \langle g \rangle$. □

PROPOSITION 579. $\langle f \rangle \bot^{\mathfrak{F}(\operatorname{Src} f)} = \bot^{\mathfrak{F}(\operatorname{Dst} f)}$ for every funcoid f.

PROOF. $\mathcal{Y} \not\asymp \langle f \rangle \bot^{\mathfrak{F}(\operatorname{Src} f)} = \bot^{\mathfrak{F}(\operatorname{Src} f)} \not\asymp \langle f^{-1} \rangle \mathcal{Y} \Leftrightarrow 0 \Leftrightarrow \mathcal{Y} \not\asymp \bot^{\mathfrak{F}(\operatorname{Dst} f)}$. Thus $\langle f \rangle \bot^{\mathfrak{F}(\operatorname{Src} f)} = \bot^{\mathfrak{F}(\operatorname{Dst} f)}$ by separability of filters. □

PROPOSITION 580. $\langle f \rangle (\mathcal{I} \sqcup \mathcal{J}) = \langle f \rangle \mathcal{I} \sqcup \langle f \rangle \mathcal{J}$ for every funcoid f and $\mathcal{I}, \mathcal{J} \in \mathfrak{F}(\operatorname{Src} f)$.

PROOF.

$$\begin{aligned}
&\star \langle f \rangle (\mathcal{I} \sqcup \mathcal{J}) = \\
&\left\{ \frac{\mathcal{Y} \in \mathfrak{F}}{\mathcal{Y} \not\asymp \langle f \rangle (\mathcal{I} \sqcup \mathcal{J})} \right\} = \\
&\left\{ \frac{\mathcal{Y} \in \mathfrak{F}}{\mathcal{I} \sqcup \mathcal{J} \not\asymp \langle f^{-1} \rangle \mathcal{Y}} \right\} = \\
&\left\{ \frac{\mathcal{Y} \in \mathfrak{F}}{\mathcal{I} \not\asymp \langle f^{-1} \rangle \mathcal{Y} \vee \mathcal{J} \not\asymp \langle f^{-1} \rangle \mathcal{Y}} \right\} = \\
&\left\{ \frac{\mathcal{Y} \in \mathfrak{F}}{\mathcal{Y} \not\asymp \langle f \rangle \mathcal{I} \vee \mathcal{Y} \not\asymp \langle f \rangle \mathcal{J}} \right\} = \\
&\left\{ \frac{\mathcal{Y} \in \mathfrak{F}}{\mathcal{Y} \not\asymp \langle f \rangle \mathcal{I} \sqcup \langle f \rangle \mathcal{J}} \right\} = \\
&\star (\langle f \rangle \mathcal{I} \sqcup \langle f \rangle \mathcal{J}).
\end{aligned}$$

Thus $\langle f \rangle (\mathcal{I} \sqcup \mathcal{J}) = \langle f \rangle \mathcal{I} \sqcup \langle f \rangle \mathcal{J}$ because $\mathfrak{F}(\operatorname{Dst} f)$ is separable. □

PROPOSITION 581. For every $f \in \mathsf{FCD}(A; B)$ for every sets A and B we have:

1°. $\mathcal{K} \; [f] \; \mathcal{I} \sqcup \mathcal{J} \Leftrightarrow \mathcal{K} \; [f] \; \mathcal{I} \vee \mathcal{K} \; [f] \; \mathcal{J}$ for every $\mathcal{I}, \mathcal{J} \in \mathfrak{F}(B)$, $\mathcal{K} \in \mathfrak{F}(A)$.

2°. $\mathcal{I} \sqcup \mathcal{J} \; [f] \; \mathcal{K} \Leftrightarrow \mathcal{I} \; [f] \; \mathcal{K} \vee \mathcal{J} \; [f] \; \mathcal{K}$ for every $\mathcal{I}, \mathcal{J} \in \mathfrak{F}(A)$, $\mathcal{K} \in \mathfrak{F}(B)$.

PROOF.

1°.

$$\begin{aligned}
&\mathcal{K} \; [f] \; \mathcal{I} \sqcup \mathcal{J} \Leftrightarrow \\
&(\mathcal{I} \sqcup \mathcal{J}) \sqcap \langle f \rangle \mathcal{K} \neq \bot^{\mathfrak{F}(B)} \Leftrightarrow \\
&\mathcal{I} \sqcap \langle f \rangle \mathcal{K} \neq \bot^{\mathfrak{F}(B)} \vee \mathcal{J} \sqcap \langle f \rangle \mathcal{K} \neq \bot^{\mathfrak{F}(B)} \Leftrightarrow \\
&\mathcal{K} \; [f] \; \mathcal{I} \vee \mathcal{K} \; [f] \; \mathcal{J}.
\end{aligned}$$

2°. Similar.

□

6.2.1. Composition of funcoids.

DEFINITION 582. Funcoids f and g are *composable* when $\operatorname{Dst} f = \operatorname{Src} g$.

DEFINITION 583. *Composition* of composable funcoids is defined by the formula

$$(B; C; \alpha_2; \beta_2) \circ (A; B; \alpha_1; \beta_1) = (A; C; \alpha_2 \circ \alpha_1; \beta_1 \circ \beta_2).$$

PROPOSITION 584. If f, g are composable funcoids then $g \circ f$ is a funcoid.