(where $\langle p\rangle^*$ is defined in the introduction), recall that a funcoid is uniquely determined by the values of its first component on sets. I will call such funcoids *principal*. So funcoids are a generalization of binary relations.

Composition of binary relations (i.e. of principal funcoids) complies with the formulas:

$$\langle g\circ f\rangle^* = \langle g\rangle^* \circ \langle f\rangle^* \quad \text{and} \quad \left\langle (g\circ f)^{-1}\right\rangle^* = \left\langle f^{-1}\right\rangle^* \circ \left\langle g^{-1}\right\rangle^*.$$

By similar formulas we can define composition of every two funcoids. Funcoids with this composition form a category (*the category of funcoids*).

Also funcoids can be reversed (like reversal of X and Y in a binary relation) by the formula $(\alpha;\beta)^{-1} = (\beta;\alpha)$. In the particular case if μ is a proximity we have $\mu^{-1} = \mu$ because proximities are symmetric.

Funcoids behave similarly to (multivalued) functions but acting on filters instead of acting on sets. Below these will be defined domain and image of a funcoid (the domain and the image of a funcoid are filters).

6.2. Basic definitions

DEFINITION 568. Let us call a *funcoid* from a set A to a set B a quadruple $(A;B;\alpha;\beta)$ where $\alpha\in\mathfrak{F}(B)^{\mathfrak{F}(A)}$, $\alpha\in\mathfrak{F}(A)^{\mathfrak{F}(B)}$ such that

$$\forall\mathcal{X}\in\mathfrak{F}(A),\mathcal{Y}\in\mathfrak{F}(B) : (\mathcal{Y}\not\asymp\alpha\mathcal{X}\Leftrightarrow\mathcal{X}\not\asymp\beta\mathcal{Y}).$$

Further we will assume that all funcoids in consideration are small without mentioning it explicitly.FiXme: It is superfluous.

DEFINITION 569. *Source* and *destination* of every funcoid $(A;B;\alpha;\beta)$ are defined as:

$$\operatorname{Src}(A;B;\alpha;\beta) = A \quad \text{and} \quad \operatorname{Dst}(A;B;\alpha;\beta) = B.$$

I will denote $\mathsf{FCD}(A;B)$ the set of funcoids from A to B.

I will denote FCD the set of all funcoids (for small sets).

DEFINITION 570. $\langle(A;B;\alpha;\beta)\rangle \overset{\text{def}}{=} \alpha$ for a funcoid $(A;B;\alpha;\beta)$.

DEFINITION 571. The *reverse* funcoid $(A;B;\alpha;\beta)^{-1} = (B;A;\beta;\alpha)$ for a funcoid $(A;B;\alpha;\beta)$.

NOTE 572. The reverse funcoid is *not* an inverse in the sense of group theory or category theory.

PROPOSITION 573. If f is a funcoid then f^{-1} is also a funcoid.

PROOF. It follows from symmetry in the definition of funcoid. □

OBVIOUS 574. $(f^{-1})^{-1} = f$ for a funcoid f.

DEFINITION 575. The relation $[f]\in\mathscr{P}(\mathfrak{F}(\operatorname{Src} f)\times\mathfrak{F}(\operatorname{Dst} f))$ is defined (for every funcoid f and $\mathcal{X}\in\mathfrak{F}(\operatorname{Src} f)$, $\mathcal{Y}\in\mathfrak{F}(\operatorname{Dst} f)$ by the formula $\mathcal{X}\;[f]\;\mathcal{Y}\Leftrightarrow\mathcal{Y}\not\asymp\langle f\rangle\mathcal{X}$.

OBVIOUS 576. $\mathcal{X}\;[f]\;\mathcal{Y}\Leftrightarrow\mathcal{Y}\not\asymp\langle f\rangle\mathcal{X}\Leftrightarrow\mathcal{X}\not\asymp\left\langle f^{-1}\right\rangle\mathcal{Y}$ for every funcoid f and $\mathcal{X}\in\mathfrak{F}(\operatorname{Src} f)$, $\mathcal{Y}\in\mathfrak{F}(\operatorname{Dst} f)$.

OBVIOUS 577. $\left[f^{-1}\right]=[f]^{-1}$ for a funcoid f.

THEOREM 578. Let A, B be sets.

1°. For given value of $\langle f\rangle\in\mathfrak{F}(B)^{\mathfrak{F}(A)}$ there exists no more than one funcoid $f\in\mathsf{FCD}(A;B)$.
2°. For given value of $[f]\in\mathscr{P}(\mathfrak{F}(A)\times\mathfrak{F}(B))$ there exists no more than one funcoid $f\in\mathsf{FCD}(A;B)$.