If $\left\{ \frac{\bigsqcup^{\mathfrak{F}} X}{X \in \mathscr{P} S} \right\}$ is closed under arbitrary meets and joins, then it is the complete lattice generated by the set S because it cannot be smaller than the set of all suprema of subsets of S.

That $\left\{ \frac{\bigsqcup^{\mathfrak{F}} X}{X \in \mathscr{P} S} \right\}$ is closed under arbitrary joins is trivial. I have not succeeded to prove that it is closed under arbitrary meets, but have proved a weaker statement that is is closed under finite meets: □

PROPOSITION 528. $\left\{ \frac{\bigsqcup^{\mathfrak{F}} X}{X \in \mathscr{P} S} \right\}$ is closed under finite meets.

PROOF. Let $R = \left\{ \frac{\bigsqcup^{\mathfrak{F}} X}{X \in \mathscr{P} S} \right\}$. Then for every $X, Y \in \mathscr{P} S$

$$\begin{aligned}
\bigsqcup^{\mathfrak{F}} X \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y &= \\
\bigsqcup^{\mathfrak{F}} ((X \cap Y) \cup (X \setminus Y)) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y &= \\
\left(\bigsqcup^{\mathfrak{F}} (X \cap Y) \sqcup^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} (X \setminus Y) \right) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y &= \\
\left(\bigsqcup^{\mathfrak{F}} (X \cap Y) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y \right) \sqcup^{\mathfrak{F}} \left(\bigsqcup^{\mathfrak{F}} (X \setminus Y) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y \right) &= \\
\left(\bigsqcup^{\mathfrak{F}} (X \cap Y) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y \right) \sqcup^{\mathfrak{F}} \bot^{\mathfrak{F}} &= \\
\bigsqcup^{\mathfrak{F}} (X \cap Y) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y.
\end{aligned}$$

Applying the formula $\bigsqcup^{\mathfrak{F}} X \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y = \bigsqcup^{\mathfrak{F}} (X \cap Y) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y$ twice we get

$$\begin{aligned}
\bigsqcup^{\mathfrak{F}} X \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} Y &= \\
\bigsqcup^{\mathfrak{F}} (X \cap Y) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} (Y \cap (X \cap Y)) &= \\
\bigsqcup^{\mathfrak{F}} (X \cap Y) \sqcap^{\mathfrak{F}} \bigsqcup^{\mathfrak{F}} (X \cap Y) &= \\
\bigsqcup^{\mathfrak{F}} (X \cap Y).
\end{aligned}$$

But for any $A, B \in R$ there exist $X, Y \in \mathscr{P} S$ such that $A = \bigsqcup^{\mathfrak{F}} X$, $B = \bigsqcup^{\mathfrak{F}} Y$. So $A \sqcap^{\mathfrak{F}} B = \bigsqcup^{\mathfrak{F}} X \sqcap \bigsqcup^{\mathfrak{F}} Y = \bigsqcup^{\mathfrak{F}} (X \cap Y) \in R$. □

4.6.2. Quasidifference.

CONJECTURE 529. $a \setminus^* b = \bigsqcup \left\{ \frac{a \sqcap \uparrow (U \setminus B)}{B \in b} \right\}$ for all $a, b \in \mathfrak{F}$ for each lattice $\mathfrak{F}$ of filters on a set U.

4.6.3. Non-Formal Problems. Find a common generalization of two theorems:

1°. If $\mathfrak{Z}$ is a meet-semilattice with greatest element then for any $\mathcal{A}, \mathcal{B} \in \mathfrak{F}$

$$\mathcal{A} \sqcup^{\mathfrak{F}} \mathcal{B} = \mathcal{A} \cap \mathcal{B}.$$