

PROPOSITION 486. $\text{Cor } a = \uparrow \left\{ \frac{p \in \mathfrak{U}}{\uparrow \{p\} \sqsubseteq a} \right\}$ and $\bigcap a = \left\{ \frac{p \in \mathfrak{U}}{\uparrow \{p\} \sqsubseteq a} \right\}$ for every filter a on a set.

PROOF. By propositions 415 and 448. \square

PROPOSITION 487. For every filter a on a set $a^* = a^+ = \overline{\text{Cor } a} = \overline{\text{Cor}' a}$.

PROOF. By propositions 416 and 448. \square

COROLLARY 488. For every filter a on a set $a^* = a^+ \in \mathfrak{F}$.

PROPOSITION 489. If a is a filter on a set, then a^+ is dual pseudocomplement of a , that is

$$a^+ = \min \left\{ \frac{c \in \mathfrak{F}}{c \sqcup^{\mathfrak{F}} a = \top^{\mathfrak{F}}} \right\}.$$

PROOF. By proposition 418. \square

PROPOSITION 490. If a, b are filters on a set, then

- 1°. $\bigcap (a \sqcap^{\mathfrak{F}} b) = \bigcap a \cap \bigcap b$;
- 2°. $\bigcap (a \sqcup^{\mathfrak{F}} b) = \bigcap a \cup \bigcap b$.

PROOF. By propositions 420 and 423. \square

PROPOSITION 491. $\bigcap \bigcap^{\mathfrak{F}} S = \bigcap (\bigcap)^* S$.

PROOF. By proposition 421. \square

PROPOSITION 492. If a, b are filters on a set, then

- 1°. $(a \sqcap^{\mathfrak{F}} b)^* = a^* \sqcup^{\mathfrak{F}} b^*$;
- 2°. $(a \sqcup^{\mathfrak{F}} b)^* = a^* \sqcap^{\mathfrak{F}} b^*$.

PROOF. By propositions 424 and 425. \square

PROPOSITION 493. For every $X, Y \in \mathcal{P}\mathfrak{U}$ and filter \mathcal{F} on \mathfrak{U} we have:

$$\uparrow X \sim \uparrow Y \Leftrightarrow \exists A \in \mathcal{A} : X \cap A = Y \cap A.$$

PROOF. By theorem 429. \square

PROPOSITION 494. Let \mathfrak{F} be the set of filters on a set \mathfrak{U} and $\mathcal{A} \in \mathfrak{F}$. Consider the function $\gamma : Z(D\mathcal{A}) \rightarrow (\mathcal{P}\mathfrak{U})/\sim$ defined by the formula (for every $p \in Z(D\mathcal{A})$)

$$\gamma p = \left\{ \frac{X \in \mathfrak{F}}{\uparrow X \sqcap^{\mathfrak{F}} \mathcal{A} = p} \right\}.$$

Then:

- 1°. γ is a lattice isomorphism.
- 2°. $\forall Q \in q : \gamma^{-1} q = \uparrow Q \sqcap^{\mathfrak{F}} \mathcal{A}$ for every $q \in (\mathcal{P}\mathfrak{U})/\sim$.

PROOF. By theorem 432. \square

PROPOSITION 495. $(\mathcal{P}\mathfrak{U})/\sim$ is a boolean lattice.

PROOF. By corollary 433. \square

PROPOSITION 496. For a lattice \mathfrak{F} of filters on a set and $a, b \in \mathfrak{F}$ the following expressions are always equal:

- 1°. $a \setminus^* b = \bigcap \left\{ \frac{z \in \mathfrak{F}}{a \sqsubseteq b \sqcup z} \right\}$ (quasidifference of a and b);
- 2°. $a \# b = \left\{ \frac{z \in \mathfrak{F}}{z \sqsubseteq a \wedge z \sqcap b = \perp} \right\}$ (second quasidifference of a and b);
- 3°. $\bigsqcup (\text{atoms } a \setminus \text{atoms } b)$.

PROOF. Theorem 434. \square