4.3.22. Pseudodifference of filters.

PROPOSITION 434. For a lattice $\mathfrak{F}$ of filters over a boolean lattice and $a, b \in \mathfrak{F}$ the following expressions are always equal:

1°. $a \setminus^* b = \bigcap \left\{ \frac{z \in \mathfrak{F}}{a \sqsubseteq b \sqcup z} \right\}$ (quasidifference of a and b);
2°. $a \# b = \bigsqcup \left\{ \frac{z \in \mathfrak{F}}{z \sqsubseteq a \wedge z \sqcap b = \bot} \right\}$ (second quasidifference of a and b);
3°. $\bigsqcup(\operatorname{atoms} a \setminus \operatorname{atoms} b)$.

PROOF. Theorem 202, taking into account corollary 382 theorem 404. □

4.4. Filters on a Set

In this section we will consider filters on the poset $\mathfrak{Z} = \mathscr{P}\mathfrak{U}$ (where $\mathfrak{U}$ is some fixed set) with the order $A \sqsubseteq B \Leftrightarrow A \subseteq B$ (for $A, B \in \mathscr{P}\mathfrak{A}$).

In fact, it is a complete atomistic boolean lattice with $\bigsqcap S = \bigcap S$, $\bigsqcup S = \bigcup S$, $\overline{A} = \mathfrak{U} \setminus A$ for every $S \in \mathscr{P}\mathscr{P}\mathfrak{U}$ and $A \in \mathscr{P}\mathfrak{U}$, atoms being one-element sets.

DEFINITION 435. I will call a filter on the lattice of all subsets of a given set $\mathfrak{U}$ as a *filter on set*.

DEFINITION 436. I will denote the set on which a filter $\mathcal{F}$ is defined as $\operatorname{Base}(\mathcal{F})$.

OBVIOUS 437. $\operatorname{Base}(\mathcal{F}) = \bigcup \mathcal{F}$.

DEFINITION 438. I will call the primary filtrator for $\mathfrak{Z} = \mathscr{P}\mathfrak{U}$ (with order on $\mathfrak{Z}$ defined as $A \sqsubseteq B \Leftrightarrow A \subseteq B$) for some set $\mathfrak{U}$ as *powerset filtrator*.

PROPOSITION 439. The following are equivalent for a non-empty set $F \in \mathscr{P}\mathscr{P}\mathfrak{U}$:

1°. F is a filter.
2°. $\forall X, Y \in F : X \cap Y \in F$ and F is an upper set.
3°. $\forall X, Y \in \mathscr{P}\mathfrak{U} : (X, Y \in F \Leftrightarrow X \cap Y \in F)$.

PROOF. By theorem 349. □

OBVIOUS 440. The minimal filter on $\mathscr{P}\mathfrak{U}$ is $\mathscr{P}\mathfrak{U}$.

OBVIOUS 441. The maximal filter on $\mathscr{P}\mathfrak{U}$ is $\{\mathfrak{U}\}$.

I will denote $\uparrow A = \uparrow^{\mathfrak{U}} A = \uparrow^{\mathscr{P}\mathfrak{U}} A$. (The distinction between conflicting notations $\uparrow^{\mathfrak{U}} A$ and $\uparrow^{\mathscr{P}\mathfrak{U}} A$ will be clear from the context.)

PROPOSITION 442. The powerset filtrator is both up-aligned and down-aligned.

PROOF. By theorem 365. □

PROPOSITION 443. Every powerset filtrator is filtered.

PROOF. By corollary 362. □

PROPOSITION 444. Every powerset filtrator is with join-closed core.

PROOF. By corollary 363. □

PROPOSITION 445. Every powerset filtrator is with finitely meet-closed core.

PROOF. By proposition 364. □

PROPOSITION 446. Every powerset filtrator is with separable core.

PROOF. By theorem 379. □

PROPOSITION 447. Every powerset filtrator is with co-separable core.