

PROPOSITION 417. Let  $\mathfrak{Z}$  be a complete lattice. Then  $a^* \in \mathfrak{P}$ .

PROOF.  $\mathfrak{F}$  is a complete lattice by 374.  $(\mathfrak{F}; \mathfrak{P})$  is a filtrator with join-closed core by corollary 363.  $(\mathfrak{F}; \mathfrak{P})$  is a filtrator with separable core by theorem 379. So we can apply theorem 331.  $\square$

PROPOSITION 418. If  $\mathfrak{Z}$  is a complete boolean lattice, then  $a^+$  is dual pseudo-complement of  $a$ , that is

$$a^+ = \min \left\{ \frac{c \in \mathfrak{A}}{c \sqcup^{\mathfrak{F}} a = \top^{\mathfrak{F}}} \right\}$$

for every  $a \in \mathfrak{F}$ .

PROOF.  $(\mathfrak{F}; \mathfrak{P})$  is filtered by the corollary 362. It is with co-separable core by theorem 340.  $\mathfrak{F}$  is a complete lattice by corollary 374. So we can apply theorem 332.  $\square$

PROPOSITION 419. For a primary filtrator over a complete boolean lattice both edge part and dual edge part are always defined.

PROOF. Core part and dual core part are defined because the core is a complete lattice. Using the theorem 383.  $\square$

PROPOSITION 420. If  $\mathfrak{Z}$  is a complete lattice, then for every  $a, b \in \mathfrak{F}$

$$\text{Cor}(a \sqcap^{\mathfrak{F}} b) = \text{Cor } a \sqcap^{\mathfrak{P}} \text{Cor } b.$$

PROOF.  $(\mathfrak{F}; \mathfrak{P})$  is with join-closed core by corollary 363.  $\mathfrak{F}$  is a meet-semilattice by corollary 374. So we can apply theorem 335. Then apply proposition 367.  $\square$

PROPOSITION 421. If  $\mathfrak{Z}$  is a complete lattice, then for every  $S \in \mathcal{P}\mathfrak{F}$

$$\text{Cor} \prod^{\mathfrak{F}} S = \prod^{\mathfrak{P}} (\text{Cor})^* S.$$

PROOF. By theorem 336.  $\square$

COROLLARY 422. If  $\mathfrak{Z}$  is a complete lattice, then for every  $S \in \mathcal{P}\mathfrak{P}$

$$\text{Cor} \prod^{\mathfrak{F}} S = \prod^{\mathfrak{P}} S.$$

PROPOSITION 423. Let  $\mathfrak{Z}$  be a complete atomistic lattice. Then for every  $a, b \in \mathfrak{F}$

$$\text{Cor}(a \sqcup^{\mathfrak{F}} b) = \text{Cor } a \sqcup^{\mathfrak{P}} \text{Cor } b.$$

PROOF.  $(\mathfrak{F}; \mathfrak{P})$  is semifiltered by corollary 362. It is with finitely meet-close core by 364.  $\mathfrak{F}$  is starrish by corollary 381.  $\mathfrak{F}$  is complete by corollary 374. So we can apply theorem 338. Then apply proposition 367.  $\square$

THEOREM 424. Let  $\mathfrak{Z}$  be a complete boolean lattice. Then  $(a \sqcap^{\mathfrak{F}} b)^* = a^* \sqcup^{\mathfrak{P}} b^*$  for every  $a, b \in \mathfrak{F}$ .

PROOF.  $(\mathfrak{F}; \mathfrak{P})$  is a filtered (corollary 362) up-aligned complete lattice filtrator with finitely join-closed (theorem 292) co-separable core (theorem 340) which is a complete boolean lattice. Thus by the theorem 327

$$(a \sqcap^{\mathfrak{F}} b)^* = (a \sqcap^{\mathfrak{F}} b)^+ = \overline{\text{Cor}(a \sqcap^{\mathfrak{F}} b)} = \overline{\text{Cor } a \sqcap^{\mathfrak{P}} \text{Cor } b} = \overline{\text{Cor } a \sqcup^{\mathfrak{F}} \text{Cor } b} = a^+ \sqcup^{\mathfrak{P}} b^+ = a^* \sqcup^{\mathfrak{P}} b^* \blacksquare$$

(used propositions 416, 420).  $\square$

THEOREM 425. Let  $\mathfrak{Z}$  be a complete atomistic boolean lattice. Then  $(a \sqcup^{\mathfrak{F}} b)^* = a^* \sqcap^{\mathfrak{P}} b^*$  for every  $a, b \in \mathfrak{F}$ .