account properties of generalized filter bases)

$$\begin{aligned}
\mathcal{X} \in S &\Leftrightarrow \\
\operatorname{up}\mathcal{X} \subseteq S &\Leftrightarrow \\
\operatorname{up}\mathcal{X} \subseteq \partial\mathcal{F} &\Leftrightarrow \\
\forall X \in \operatorname{up}\mathcal{X} : X \sqcap^{\mathfrak{F}} \mathcal{F} \neq \bot^{\mathfrak{F}} &\Leftrightarrow \\
\bot^{\mathfrak{F}} \notin \langle \mathcal{F} \sqcap^{\mathfrak{F}} \rangle^* \operatorname{up}\mathcal{X} &\Leftrightarrow \\
\bigsqcap^{\mathfrak{F}} \langle \mathcal{F} \sqcap^{\mathfrak{F}} \rangle^* \operatorname{up}\mathcal{X} \neq \bot^{\mathfrak{F}} &\Leftrightarrow \\
\mathcal{F} \sqcap^{\mathfrak{F}} \bigsqcap^{\mathfrak{F}} \operatorname{up}\mathcal{X} \neq \bot^{\mathfrak{F}} &\Leftrightarrow \\
\mathcal{F} \sqcap^{\mathfrak{F}} \mathcal{X} \neq \bot^{\mathfrak{F}} &\Leftrightarrow \\
\mathcal{X} \in \star\mathcal{F}.
\end{aligned}$$

□

4.3.18. Filters and a Special Sublattice.

THEOREM 414. Let $(\mathfrak{F};\mathfrak{Z})$ be a primary filtrator where $\mathfrak{Z}$ is a boolean lattice. Let $\mathcal{A} \in \mathfrak{F}$. Then for each $\mathcal{X} \in \mathfrak{F}$

$$\mathcal{X} \in Z(D\mathcal{A}) \Leftrightarrow \exists X \in \mathfrak{P} : \mathcal{X} = X \sqcap^{\mathfrak{F}} \mathcal{A}.$$

PROOF.

$\Leftarrow$. Let $\mathcal{X} = X \sqcap^{\mathfrak{F}} \mathcal{A}$ where $X \in \mathfrak{P}$. Let also $\mathcal{Y} = \overline{X} \sqcap^{\mathfrak{F}} \mathcal{A}$. Then $\mathcal{X} \sqcap^{\mathfrak{F}} \mathcal{Y} = X \sqcap^{\mathfrak{F}} \overline{X} \sqcap^{\mathfrak{F}} \mathcal{A} = (X \sqcap^{\mathfrak{P}} \overline{X}) \sqcap^{\mathfrak{F}} \mathcal{A} = \bot^{\mathfrak{F}} \sqcap^{\mathfrak{F}} \mathcal{A} = \bot^{\mathfrak{F}}$ (used theorem 311) and $\mathcal{X} \sqcup^{\mathfrak{F}} \mathcal{Y} = (X \sqcup^{\mathfrak{F}} \overline{X}) \sqcap^{\mathfrak{F}} \mathcal{A} = (X \sqcup^{\mathfrak{P}} \overline{X}) \sqcap^{\mathfrak{F}} \mathcal{A} = \top^{\mathfrak{F}} \sqcap^{\mathfrak{F}} \mathcal{A} = \mathcal{A}$ (used the theorems 292 and corollary 381). So $\mathcal{X} \in Z(D\mathcal{A})$.

$\Rightarrow$. Let $\mathcal{X} \in Z(D\mathcal{A})$. Then there exists $\mathcal{Y} \in Z(D\mathcal{A})$ such that $\mathcal{X} \sqcap^{\mathfrak{F}} \mathcal{Y} = \bot^{\mathfrak{F}}$ and $\mathcal{X} \sqcup^{\mathfrak{F}} \mathcal{Y} = \mathcal{A}$. Then (used theorem 379) there exists $X \in \operatorname{up}\mathcal{X}$ such that $X \sqcap^{\mathfrak{F}} \mathcal{Y} = \bot^{\mathfrak{F}}$. We have

$$\mathcal{X} = \mathcal{X} \sqcup (X \sqcap^{\mathfrak{F}} \mathcal{Y}) = \mathcal{X} = X \sqcap^{\mathfrak{F}} (\mathcal{X} \sqcap^{\mathfrak{F}} \mathcal{Y}) = X \sqcap^{\mathfrak{F}} \mathcal{A}.$$

□

4.3.19. Core Part and Atomic Elements.

PROPOSITION 415. Let $\mathfrak{Z}$ be an atomistic lattice. Then for every $a \in \mathfrak{F}$ such that $\operatorname{Cor}' a$ exists we have

$$\operatorname{Cor}' a = \bigsqcup^{\mathfrak{Z}} \left\{ \frac{x}{x \text{ is an atom of } \mathfrak{Z}, x \sqsubseteq a} \right\}.$$

PROOF. $(\mathfrak{F};\mathfrak{P})$ is with join-closed core by corollary 363. So we can apply theorem 334. □

4.3.20. Complements and Core Parts.

PROPOSITION 416. Let $\mathfrak{Z}$ be a complete boolean lattice. Then $a^* = a^+ = \overline{\operatorname{Cor} a}$ for every $a \in \mathfrak{F}$.

PROOF. The filtrator $(\mathfrak{F};\mathfrak{P})$ is filtered by the corollary 362. $\mathfrak{F}$ is a complete lattice by corollary 374. $(\mathfrak{F};\mathfrak{P})$ is with co-separable core by theorem 340. Thus we can apply the theorem 327.

$(\mathfrak{F};\mathfrak{P})$ is filtered by the corollary 362, finitely meet-closed by proposition 364, with separable core by theorem 379. $\mathfrak{F}$ is a complete lattice by corollary 374. So we can apply the theorem 329. □