4.3.5. Alignment.

OBVIOUS 365.

1°. If $\mathfrak{Z}$ has least element, the primary filtrator is down-aligned.
2°. If $\mathfrak{Z}$ has greatest element, the primary filtrator is up-aligned.

4.3.6. Co-separability of Core for Primary Filtrators.

PROPOSITION 366. Every primary filtrator over a meet infinite distributive complete lattice is with co-separable core.

PROOF. It is up-aligned, filtered. So we can apply the theorem 340. □

4.3.7. Core Part.

PROPOSITION 367. $\operatorname{Cor}' a = \operatorname{Cor} a$ for every filter a on a complete lattice.

PROOF. By the theorem 301 and corollary 362. □

PROPOSITION 368. $\operatorname{Cor} a \sqsubseteq a$ for every filter a on a complete lattice.

PROOF. By the theorem 296 and corollary 362. □

PROPOSITION 369. $\operatorname{Cor} a = \max \operatorname{down} a$ for every filter a on a complete lattice.

PROOF. Proposition 367, obvious 302, corollary 363. □

4.3.8. Intersecting and Joining with an Element of the Core.

THEOREM 370. For a filtrator $(\mathfrak{F}; \mathfrak{P})$ where $\mathfrak{Z}$ is a boolean lattice, for every $B \in \mathfrak{P}$, $\mathcal{A} \in \mathfrak{F}$:

1°. $B \asymp^{\mathfrak{F}} \mathcal{A} \Leftrightarrow \overline{B} \sqsupseteq \mathcal{A}$;
2°. $B \equiv^{\mathfrak{F}} \mathcal{A} \Leftrightarrow B \sqsubseteq \mathcal{A}$ if $\mathfrak{Z}$ is a complete lattice.

PROOF.

1°. Using theorem 310, obvious 365, proposition 364, theorem 379.FiXme: Forward reference!
2°. Using theorem 310, obvious 365, corollary 363, theorem 340.

□

4.3.9. Formulas for Meets and Joins of Filters.

LEMMA 371. If f is an order embedding from a poset $\mathfrak{A}$ to a complete lattice $\mathfrak{B}$ and $S \in \mathscr{P}\mathfrak{A}$ and there exists such $\mathcal{F} \in \mathfrak{A}$ that $f\mathcal{F} = \bigsqcup^{\mathfrak{B}} \langle f \rangle^* S$, then $\bigsqcup^{\mathfrak{A}} S$ exists and $f \bigsqcup^{\mathfrak{A}} S = \bigsqcup^{\mathfrak{B}} \langle f \rangle^* S$.

PROOF. f is an order isomorphism from $\mathfrak{A}$ to $\mathfrak{B}|_{\langle f \rangle^* \mathfrak{A}}$. $f\mathcal{F} \in \mathfrak{B}|_{\langle f \rangle^* \mathfrak{A}}$.
Consequently, $\bigsqcup^{\mathfrak{B}} \langle f \rangle^* S \in \mathfrak{B}|_{\langle f \rangle^* \mathfrak{A}}$ and $\bigsqcup^{\mathfrak{B}|_{\langle f \rangle^* \mathfrak{A}}} \langle f \rangle^* S = \bigsqcup^{\mathfrak{B}} \langle f \rangle^* S$.
$f \bigsqcup^{\mathfrak{A}} S = \bigsqcup^{\mathfrak{B}|_{\langle f \rangle^* \mathfrak{A}}} \langle f \rangle^* S$ because f is an order isomorphism.
Combining, $f \bigsqcup^{\mathfrak{A}} S = \bigsqcup^{\mathfrak{B}} \langle f \rangle^* S$. □

COROLLARY 372. If $\mathfrak{B}$ is a complete lattice and $\mathfrak{A}$ is its subset and $S \in \mathscr{P}\mathfrak{A}$, then $\bigsqcup^{\mathfrak{A}} S$ exists and $\bigsqcup^{\mathfrak{A}} S = \bigsqcup^{\mathfrak{B}} S$.

THEOREM 373. If $\mathfrak{Z}$ is a meet-semilattice with greatest element $\top$ then $\bigsqcup^{\mathfrak{F}} S$ exists and

$$\bigsqcup^{\mathfrak{F}} S = \bigcap S$$

for every $S \in \mathscr{P}\mathfrak{F}$.