4.2.9. Complements and Core Parts.

LEMMA 326. If $(\mathfrak{A};\mathfrak{Z})$ is a filtered, up-aligned filtrator with co-separable core which is a complete lattice, then for any $a, c \in \mathfrak{A}$

$$c \equiv^{\mathfrak{A}} a \Leftrightarrow c \equiv^{\mathfrak{A}} \operatorname{Cor} a.$$

PROOF.

$\Rightarrow$. If $c \equiv^{\mathfrak{A}} a$ then by co-separability of the core exists $K \in \operatorname{down} a$ such that $c \equiv^{\mathfrak{A}} K$. To finish the proof we will show that $K \sqsubseteq \operatorname{Cor} a$. To show this is enough to show that $\forall X \in \operatorname{up} a : K \sqsubseteq X$ what is obvious.

$\Leftarrow$. $\operatorname{Cor} a \sqsubseteq a$ (by the theorem 296 using that our filtrator is filtered).

□

THEOREM 327. If $(\mathfrak{A};\mathfrak{Z})$ is a filtered up-aligned complete lattice filtrator with co-separable core which is a complete boolean lattice, then $a^+ = \overline{\operatorname{Cor} a}$ for every $a \in \mathfrak{A}$.

PROOF. Our filtrator is with join-closed core (theorem 292).

$$a^+ = \bigsqcap^{\mathfrak{A}} \left\{ \frac{c \in \mathfrak{A}}{c \sqcup^{\mathfrak{A}} a = \top^{\mathfrak{A}}} \right\} = \bigsqcap^{\mathfrak{A}} \left\{ \frac{c \in \mathfrak{A}}{c \sqcup^{\mathfrak{A}} \operatorname{Cor} a = \top^{\mathfrak{A}}} \right\} = \bigsqcap^{\mathfrak{A}} \left\{ \frac{c \in \mathfrak{A}}{c \sqsupseteq \overline{\operatorname{Cor} a}} \right\} = \overline{\operatorname{Cor} a}$$

(used the lemma and theorem 310). □

COROLLARY 328. If $(\mathfrak{A};\mathfrak{Z})$ is a filtered up-aligned complete lattice filtrator with co-separable core which is a complete boolean lattice, then $a^+ \in \mathfrak{Z}$ for every $a \in \mathfrak{A}$.

THEOREM 329. If $(\mathfrak{A};\mathfrak{Z})$ is a filtered complete lattice filtrator with down-aligned, finitely meet-closed, separable core which is a complete boolean lattice, then $a^* = \overline{\operatorname{Cor} a} = \overline{\operatorname{Cor}' a}$ for every $a \in \mathfrak{A}$.