PROOF. $\operatorname{Cor} a = \bigsqcap^{\mathfrak{Z}} \operatorname{up} a \sqsupseteq \operatorname{Cor}' a$ because $\forall A \in \operatorname{up} a : \operatorname{Cor}' a \sqsubseteq A$. □

THEOREM 301. $\operatorname{Cor}' a = \operatorname{Cor} a$ whenever both $\operatorname{Cor} a$ and $\operatorname{Cor}' a$ exist for any element a of a filtered filtrator.

PROOF. It is with join-closed core because it is semifiltered. So $\operatorname{Cor}' a \sqsubseteq \operatorname{Cor} a$. $\operatorname{Cor} a \in \operatorname{down} a$. So $\operatorname{Cor} a \sqsubseteq \bigsqcup^{\mathfrak{Z}} \operatorname{down} a = \operatorname{Cor}' a$. □

OBVIOUS 302. $\operatorname{Cor}' a = \max \operatorname{down} a$ for an element a of a filtrator with join-closed core.

4.2.2. Filtrators with Separable Core.

DEFINITION 303. Let $(\mathfrak{A}; \mathfrak{Z})$ be a filtrator. It is a *filtrator with separable core* when

$$\forall x, y \in \mathfrak{A} : (x \asymp^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{up} x : X \asymp^{\mathfrak{A}} y).$$

PROPOSITION 304. Let $(\mathfrak{A}; \mathfrak{Z})$ be a filtrator. It is a *filtrator with separable core* iff

$$\forall x, y \in \mathfrak{A} : (x \asymp^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{up} x, Y \in \operatorname{up} y : X \asymp^{\mathfrak{A}} Y).$$

PROOF.

$\Rightarrow$. Apply the definition twice.
$\Leftarrow$. Obvious.

□

DEFINITION 305. Let $(\mathfrak{A}; \mathfrak{Z})$ be a filtrator. It is a *filtrator with co-separable core* when

$$\forall x, y \in \mathfrak{A} : (x \equiv^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{down} x : X \equiv^{\mathfrak{A}} y).$$

OBVIOUS 306. Co-separability is the dual of separability.

DEFINITION 307. Let $(\mathfrak{A}; \mathfrak{Z})$ be a filtrator. It is a *filtrator with co-separable core* when

$$\forall x, y \in \mathfrak{A} : (x \equiv^{\mathfrak{A}} y \Rightarrow \exists X \in \operatorname{down} x, Y \in \operatorname{down} y : X \equiv^{\mathfrak{A}} X).$$

PROOF. By duality. □

4.2.3. Intersection and Joining with an Element of the Core.

DEFINITION 308. I call *down-aligned* filtrator such a filtrator $(\mathfrak{A}; \mathfrak{Z})$ that $\mathfrak{A}$ and $\mathfrak{Z}$ have common least element. (Let's denote it $\bot$.)

DEFINITION 309. I call *up-aligned* filtrator such a filtrator $(\mathfrak{A}; \mathfrak{Z})$ that $\mathfrak{A}$ and $\mathfrak{Z}$ have common greatest element. (Let's denote it $\top$.)

THEOREM 310. For a filtrator $(\mathfrak{A}; \mathfrak{Z})$ where $\mathfrak{Z}$ is a boolean lattice, for every $B \in \mathfrak{Z}$, $\mathcal{A} \in \mathfrak{A}$:

1°. $B \asymp^{\mathfrak{A}} \mathcal{A} \Leftrightarrow \overline{B} \sqsupseteq \mathcal{A}$ if it is down-aligned, with finitely meet-closed and separable core;
2°. $B \equiv^{\mathfrak{A}} \mathcal{A} \Leftrightarrow \overline{B} \sqsubseteq \mathcal{A}$ if it is up-aligned, with finitely join-closed and co-separable core.