Definition 284. I call a filtrator *with meet-closed core* such a filtrator $(\mathfrak{A};\mathfrak{Z})$ that $\bigsqcap^{\mathfrak{Z}} S = \bigsqcap^{\mathfrak{A}} S$ whenever $\bigsqcap^{\mathfrak{Z}} S$ exists for $S \in \mathscr{P}\mathfrak{Z}$.

Definition 285. I call a filtrator *with finitely join-closed core* such a filtrator $(\mathfrak{A};\mathfrak{Z})$ that $a \sqcup^{\mathfrak{Z}} b = a \sqcup^{\mathfrak{A}} b$ whenever $a \sqcup^{\mathfrak{Z}} b$ exists for $a, b \in \mathfrak{Z}$.

Definition 286. I call a filtrator *with finitely meet-closed core* such a filtrator $(\mathfrak{A};\mathfrak{Z})$ that $a \sqcap{}^{\mathfrak{Z}} b = a \sqcap^{\mathfrak{A}} b$ whenever $a \sqcap^{\mathfrak{Z}} b$ exists for $a, b \in \mathfrak{Z}$.

Definition 287. *Filtered filtrator* is a filtrator $(\mathfrak{A};\mathfrak{Z})$ such that $\forall a \in \mathfrak{A} : a = \bigsqcap^{\mathfrak{A}} \operatorname{up} a$.

Definition 288. *Prefiltered filtrator* is a filtrator $(\mathfrak{A};\mathfrak{Z})$ such that "up" is injective.

Definition 289. *Semifiltered filtrator* is a filtrator $(\mathfrak{A};\mathfrak{Z})$ such that

$$\forall a, b \in \mathfrak{A} : (\operatorname{up} a \supseteq \operatorname{up} b \Rightarrow a \sqsubseteq b).$$

Obvious 290.
- Every filtered filtrator is semifiltered.
- Every semifiltered filtrator is prefiltered.

Obvious 291. "up" is a straight map from $\mathfrak{A}$ to the dual of the poset $\mathscr{P}\mathfrak{Z}$ if $(\mathfrak{A};\mathfrak{Z})$ is a semifiltered filtrator.

Theorem 292. Each semifiltered filtrator is a filtrator with join-closed core.

Proof. Let $(\mathfrak{A};\mathfrak{Z})$ be a semifiltered filtrator. Let $S \in \mathscr{P}\mathfrak{Z}$ and $\bigsqcup^{\mathfrak{Z}} S$ be defined. We need to prove $\bigsqcup^{\mathfrak{A}} S = \bigsqcup^{\mathfrak{Z}} S$. That $\bigsqcup^{\mathfrak{Z}} S$ is an upper bound for S is obvious. Let $a \in \mathfrak{A}$ be an upper bound for S. It's enough to prove that $\bigsqcup^{\mathfrak{Z}} S \sqsubseteq a$. Really,

$$c \in \operatorname{up} a \Rightarrow c \sqsupseteq a \Rightarrow \forall x \in S : c \sqsupseteq x \Rightarrow c \sqsupseteq \bigsqcup^{\mathfrak{Z}} S \Rightarrow c \in \operatorname{up} \bigsqcup^{\mathfrak{Z}} S;$$

so $\operatorname{up} a \subseteq \operatorname{up} \bigsqcup^{\mathfrak{Z}} S$ and thus $a \sqsupseteq \bigsqcup^{\mathfrak{Z}} S$ because it is semifiltered. □

4.2.1. Core Part.

Definition 293. The *core part* of an element $a \in \mathfrak{A}$ is $\operatorname{Cor} a = \bigsqcap^{\mathfrak{Z}} \operatorname{up} a$.

Definition 294. The *dual core part* of an element $a \in \mathfrak{A}$ is $\operatorname{Cor}' a = \bigsqcup^{\mathfrak{Z}} \operatorname{up} a$.

Obvious 295. Cor' is dual of Cor.

Theorem 296. $\operatorname{Cor} a \sqsubseteq a$ whenever $\operatorname{Cor} a$ exists for any element a of a filtered filtrator.

Proof. $\operatorname{Cor} a = \bigsqcap^{\mathfrak{Z}} \operatorname{up} a \sqsubseteq \bigsqcap^{\mathfrak{A}} \operatorname{up} a = a$. □

Corollary 297. $\operatorname{Cor} a \in \operatorname{down} a$ whenever $\operatorname{Cor} a$ exists for any element a of a filtered filtrator.

Theorem 298. $\operatorname{Cor}' a \sqsubseteq a$ whenever $\operatorname{Cor}' a$ exists for any element a of a filtrator with join-closed core.

Proof. $\operatorname{Cor}' a = \bigsqcup^{\mathfrak{Z}} \operatorname{down} a = \bigsqcup^{\mathfrak{A}} \operatorname{down} a \sqsubseteq a$. □

Corollary 299. $\operatorname{Cor}' a \in \operatorname{down} a$ whenever $\operatorname{Cor}' a$ exists for any element a of a filtrator with join-closed core.

Proposition 300. $\operatorname{Cor}' a \sqsubseteq \operatorname{Cor} a$ whenever both $\operatorname{Cor} a$ and $\operatorname{Cor}' a$ exist for any element a of a filtrator with join-closed core.