

PROOF. $\text{concat}(\llbracket x \rrbracket) = x$ for a function x taking an ordinal number of argument is obvious. It is remained to prove

$$\text{concat}(\text{concat} \circ S) = \text{concat}(\text{concat } S);$$

We have, using the lemmas, **Fixme: Say that \sim implies equality.**

$$\begin{aligned} \text{concat}(\text{concat} \circ S) &\sim \\ \text{uncurry}(\text{concat} \circ S) &\sim \\ &\text{(by lemma 263)} \\ \text{uncurry}(\text{uncurry} \circ S) &\sim \\ \text{uncurry}(\text{uncurry } S) &\sim \\ \text{uncurry}(\text{concat } S) &\sim \\ \text{concat}(\text{concat } S). & \end{aligned}$$

□

COROLLARY 266. Ordinated product is an infinitely associative function.