OBVIOUS 241. If z is a finite family of finitary tuples, it is concatenation of $\operatorname{dom} z$ tuples in the usual sense (as it is commonly used in computer science).

PROPOSITION 242. If $z \in \prod(\mathrm{GR} \circ F)$ then $\operatorname{concat} z = \operatorname{uncurry}(z) \circ (\bigoplus(\operatorname{arity} \circ F))^{-1}$.

PROOF. If $z \in \prod(\mathrm{GR} \circ F)$ then $\operatorname{dom} z(i) = \operatorname{dom}(\mathrm{GR} \circ F)_i = \operatorname{dom} F_i = \operatorname{arity} F_i$ for every $i \in \operatorname{dom} F$. Thus $\operatorname{dom} \circ z = \operatorname{arity} \circ F$. □

PROPOSITION 243. $\operatorname{dom} \operatorname{concat} z = \sum_{i \in \operatorname{dom} z} \operatorname{dom} z_i$.

PROOF. Because $\operatorname{dom}(\bigoplus(\operatorname{dom} \circ z))^{-1} = \sum_{i \in \operatorname{dom} f}(\operatorname{dom} \circ z)$, it is enough to prove that

$$\operatorname{dom} \operatorname{uncurry}(z) = \operatorname{dom} \bigoplus(\operatorname{dom} \circ z).$$

Really,

$$\sum_{i \in \operatorname{dom} f} (\operatorname{dom} \circ z) = \left\{ \frac{(i; x)}{i \in \operatorname{dom}(\operatorname{dom} \circ z), x \in \operatorname{dom} z_i} \right\} = \left\{ \frac{(i; x)}{i \in \operatorname{dom} z, x \in \operatorname{dom} z_i} \right\} = \coprod z$$

and $\operatorname{dom} \operatorname{uncurry}(z) = \coprod_{i \in X} z_i = \coprod z$. □

3.8.4.3. *Finite example.* If F is a finite family (indexed by a natural number $\operatorname{dom} F$) of anchored finitary relations, then by definition

$$\mathrm{GR} \prod^{(\mathrm{ord})} = \left\{ \frac{[\![a_{0,0}; \dots; a_{0,\operatorname{arity} F_0 - 1}; \dots; a_{\operatorname{dom} F-1,0}; \dots; a_{\operatorname{dom} F-1,\operatorname{arity} F_{\operatorname{dom} F-1}-1}]\!]}{[\![a_{0,0}; \dots; a_{0,\operatorname{arity} F_0-1}]\!] \in \mathrm{GR} F_0 \wedge \dots \wedge [\![a_{\operatorname{dom} F-1,\operatorname{arity} F_{\operatorname{dom} F-1}-1}]\!] \in \mathrm{GR} F_{\operatorname{dom} F-1}} \right\}$$

and

$$\operatorname{arity} \prod^{(\mathrm{ord})} F = \operatorname{arity} F_0 + \dots + \operatorname{arity} F_{\operatorname{dom} F-1}.$$

The above formula can be shortened to

$$\mathrm{GR} \prod^{(\mathrm{ord})} F = \left\{ \frac{\operatorname{concat} z}{z \in \prod(\mathrm{GR} \circ F)} \right\}.$$

3.8.4.4. *The definition.*

DEFINITION 244. The anchored relation (which I call *ordinated product*) $\prod^{(\mathrm{ord})} F$ is defined by the formulas:

$$\operatorname{arity} \prod^{(\mathrm{ord})} F = \sum(\operatorname{arity} \circ f);$$

$$\mathrm{GR} \prod^{(\mathrm{ord})} F = \left\{ \frac{\operatorname{concat} z}{z \in \prod(\mathrm{GR} \circ F)} \right\}.$$

PROPOSITION 245. $\prod^{(\mathrm{ord})} F$ is a properly defined anchored relation.

PROOF. $\operatorname{dom} \operatorname{concat} z = \sum_{i \in \operatorname{dom} F} \operatorname{dom} z_i = \sum_{i \in \operatorname{dom} F} \operatorname{arity} f_i = \sum(\operatorname{arity} \circ F)$. □