Monovalued. Let f and g be monovalued morphisms, $\operatorname{Dst} f = \operatorname{Src} g$. Then

$$\begin{aligned}(g \circ f) \circ (g \circ f)^\dagger &= \\ g \circ f \circ f^\dagger \circ g^\dagger &\sqsubseteq \\ g \circ 1_{\operatorname{Src} g} \circ g^\dagger &= \\ g \circ g^\dagger &\sqsubseteq \\ 1_{\operatorname{Dst} g} = 1_{\operatorname{Dst}(g \circ f)}&.\end{aligned}$$

So $g \circ f$ is monovalued.

That identity morphisms are monovalued follows from the following:

$$1_A \circ (1_A)^\dagger = 1_A \circ 1_A = 1_A = 1_{\operatorname{Dst} 1_A} \sqsubseteq 1_{\operatorname{Dst} 1_A}.$$

Entirely defined. Let f and g be entirely defined morphisms, $\operatorname{Dst} f = \operatorname{Src} g$. Then

$$\begin{aligned}(g \circ f)^\dagger \circ (g \circ f) &= \\ f^\dagger \circ g^\dagger \circ g \circ f &\sqsupseteq \\ f^\dagger \circ 1_{\operatorname{Src} g} \circ f &= \\ f^\dagger \circ 1_{\operatorname{Dst} f} \circ f &= \\ f^\dagger \circ f &\sqsupseteq \\ 1_{\operatorname{Src} f} = 1_{\operatorname{Src}(g \circ f)}&.\end{aligned}$$

So $g \circ f$ is entirely defined.

That identity morphisms are entirely defined follows from the following:

$$(1_A)^\dagger \circ 1_A = 1_A \circ 1_A = 1_A = 1_{\operatorname{Src} 1_A} \sqsupseteq 1_{\operatorname{Src} 1_A}.$$

□

Definition 218. I will call a *bijective* morphism a morphism which is entirely defined, monovalued, injective, and surjective.

Proposition 219. If a morphism is bijective then it is an isomorphism.

Proof. Let f be bijective. Then $f \circ f^\dagger \sqsubseteq 1_{\operatorname{Dst} f}$, $f^\dagger \circ f \sqsupseteq 1_{\operatorname{Src} f}$, $f^\dagger \circ f \sqsubseteq 1_{\operatorname{Src} f}$, $f \circ f^\dagger \sqsupseteq 1_{\operatorname{Dst} f}$. Thus $f \circ f^\dagger = 1_{\operatorname{Dst} f}$ and $f^\dagger \circ f = 1_{\operatorname{Src} f}$ that is $f^\dagger$ is an inverse of f. □

FiXme: Below require that Mor-sets are complete lattices.FiXme: Add meta-examples for the category **Rel**. *

Definition 220. A morphism f of a partially ordered category is *metamonovalued* when $(\bigsqcap G) \circ f = \bigsqcap_{g \in G}(g \circ f)$ whenever G is a set of morphisms with a suitable domain and image.

Definition 221. A morphism f of a partially ordered category is *metainjective* when $f \circ (\bigsqcap G) = \bigsqcap_{g \in G}(f \circ g)$ whenever G is a set of morphisms with a suitable domain and image.

Obvious 222. Metamonovaluedness and metainjectivity are dual to each other.

Definition 223. A morphism f of a partially ordered category is *metacomplete* when $f \circ (\bigsqcup G) = \bigsqcup_{g \in G}(f \circ g)$ whenever G is a set of morphisms with a suitable domain and image.

Definition 224. A morphism f of a partially ordered category is *co-metacomplete* when $(\bigsqcup G) \circ f = \bigsqcup_{g \in G}(g \circ f)$ whenever G is a set of morphisms with a suitable domain and image.