that is

$$\bigsqcup\left\{\frac{z\in\mathfrak{A}}{z\sqsubseteq a\wedge z\sqcap b=\bot}\right\}=\bigsqcup\left\{\frac{z\in\operatorname{atoms}a}{z\sqcap b=\bot}\right\}=\bigsqcup(\operatorname{atoms}a\setminus\operatorname{atoms}b).$$

□

3.5. Partially ordered categories

3.5.1. Definition.

DEFINITION 203. I will call a partially ordered (pre)category a (pre)category together with partial order $\sqsubseteq$ on each of its Mor-sets with the additional requirement that

$$f_1\sqsubseteq f_2\wedge g_1\sqsubseteq g_2\Rightarrow g_1\circ f_1\sqsubseteq g_2\circ f_2$$

for every morphisms f_1, g_1, f_2, g_2 such that $\operatorname{Src}f_1=\operatorname{Src}f_2$ and $\operatorname{Dst}f_1=\operatorname{Dst}f_2=\operatorname{Src}g_1=\operatorname{Src}g_2$ and $\operatorname{Dst}g_1=\operatorname{Dst}g_2$.

3.5.2. Dagger categories.

DEFINITION 204. FiXme: In index I sometimes use *pre-category* and sometimes *precategory*.I will call a *dagger precategory* a precategory together with an involutive contravariant identity-on-objects prefunctor $x\mapsto x^\dagger$.

In other words, a dagger precategory is a precategory equipped with a function $x\mapsto x^\dagger$ on its set of morphisms which reverses the source and the destination and is subject to the following identities for every morphisms f and g:

1°. $f^{\dagger\dagger}=f$;
2°. $(g\circ f)^\dagger=f^\dagger\circ g^\dagger$.

DEFINITION 205. I will call a *dagger category* a category together with an involutive contravariant identity-on-objects functor $x\mapsto x^\dagger$.

In other words, a dagger category is a category equipped with a function $x\mapsto x^\dagger$ on its set of morphisms which reverses the source and the destination and is subject to the following identities for every morphisms f and g and object A:

1°. $f^{\dagger\dagger}=f$;
2°. $(g\circ f)^\dagger=f^\dagger\circ g^\dagger$;
3°. $(1_A)^\dagger=1_A$.

THEOREM 206. If a category is a dagger precategory then it is a dagger category.

PROOF. We need to prove only that $(1_A)^\dagger=1_A$. Really,

$$(1_A)^\dagger=(1_A)^\dagger\circ 1_A=(1_A)^\dagger\circ(1_A)^{\dagger\dagger}=((1_A)^\dagger\circ 1_A)^\dagger=(1_A)^{\dagger\dagger}=1_A.$$

□

For a partially ordered dagger (pre)category I will additionally require (for every morphisms f and g with the same source and destination)

$$f^\dagger\sqsubseteq g^\dagger\Leftrightarrow f\sqsubseteq g.$$

An example of dagger category is the category **Rel** whose objects are sets and whose morphisms are binary relations between these sets with usual composition of binary relations and with $f^\dagger=f^{-1}$.