

2°.

$$\begin{aligned}
a \# (a \sqcap b) &= \\
\sqcup \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge z \sqcap a \sqcap b = \perp} \right\} &= \\
\sqcup \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a \wedge (z \sqcap a) \sqcap a \sqcap b = \perp} \right\} &= \\
\sqcup \left\{ \frac{z \sqcap a}{z \in \mathfrak{A}, z \sqcap a \sqcap b = \perp} \right\} &= \\
\sqcup \left\{ \frac{z \in \mathfrak{A}}{z \sqsubseteq a, z \sqcap b = \perp} \right\} &= \\
a \# b. &
\end{aligned}$$

□

I will denote Da the lattice $\left\{ \frac{x \in \mathfrak{A}}{x \sqsubseteq a} \right\}$.

THEOREM 200. For $a, b \in \mathfrak{A}$ where \mathfrak{A} is a distributive lattice with least element

1°. $a \setminus^* b = (a \sqcap b)^{+(Da)}$; **FixMe: least element is not required?**

2°. $a \# b = (a \sqcap b)^{*(Da)}$.

PROOF.

1°.

$$\begin{aligned}
(a \sqcap b)^{+(Da)} &= \\
\sqcap \left\{ \frac{c \in Da}{c \sqcup (a \sqcap b) = a} \right\} &= \\
\sqcap \left\{ \frac{c \in Da}{c \sqcup (a \sqcap b) \sqsupseteq a} \right\} &= \\
\sqcap \left\{ \frac{c \in Da}{(c \sqcup a) \sqcap (c \sqcup b) \sqsupseteq a} \right\} &= \\
\sqcap \left\{ \frac{c \in \mathfrak{A}}{c \sqsubseteq a \wedge c \sqcup b \sqsupseteq a} \right\} &= \\
a \setminus^* b. &
\end{aligned}$$

2°.

$$\begin{aligned}
(a \sqcap b)^{*(Da)} &= \\
\sqcup \left\{ \frac{c \in Da}{c \sqcap a \sqcap b = \perp} \right\} &= \\
\sqcup \left\{ \frac{c \in \mathfrak{A}}{c \sqsubseteq a \wedge c \sqcap a \sqcap b = \perp} \right\} &= \\
\sqcup \left\{ \frac{c \in \mathfrak{A}}{c \sqsubseteq a \wedge c \sqcap b = \perp} \right\} &= \\
a \# b. &
\end{aligned}$$

□

PROPOSITION 201. $(a \sqcup b) \setminus^* b \sqsubseteq a$ for an arbitrary complete lattice.

PROOF. $(a \sqcup b) \setminus^* b = \sqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqcup b \sqsubseteq b \sqcup z} \right\}$.

But $a \sqsubseteq z \Rightarrow a \sqcup b \sqsubseteq b \sqcup z$. So $\left\{ \frac{z \in \mathfrak{A}}{a \sqcup b \sqsubseteq b \sqcup z} \right\} \supseteq \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq z} \right\}$.