5°. $\forall a, b \in \mathfrak{A} : (a \sqsubset b \Rightarrow fa \not\sqsupseteq fb)$.
6°. $\forall a, b \in \mathfrak{A} : (fa \sqsubseteq fb \Rightarrow a \not\sqsupset b)$.

PROOF.

1°⇒3°. Let $a, b \in \mathfrak{A}$. Let $fa = fb \Rightarrow a = b$. Let $a \sqsubset b$. $fa \neq fb$ because $a \neq b$. $fa \sqsubseteq fb$ because $a \sqsubseteq b$. So $fa \sqsubset fb$.

2°⇒1°. Let $a, b \in \mathfrak{A}$. Let $fa \sqsubseteq fb \Rightarrow a \sqsubseteq b$. Let $fa = fb$. Then $a \sqsubseteq b$ and $b \sqsubseteq a$ and consequently $a = b$.

3°⇒2°. Let $\forall a, b \in \mathfrak{A} : (a \sqsubset b \Rightarrow fa \sqsubset fb)$. Let $a \not\sqsubseteq b$. Then $a \sqsupset a \sqcap b$. So $fa \sqsupset f(a \sqcap b)$. If $fa \sqsubseteq fb$ then $fa \sqsubseteq f(a \sqcap b)$ what is a contradiction.

3°⇒5°⇒4°. Obvious.

4°⇒3°. Because $a \sqsubset b \Rightarrow a \sqsubseteq b \Rightarrow fa \sqsubseteq fb$.

5°⇔6°. Obvious.

□

### 3.1.2. Separation subsets and full stars.

DEFINITION 166. $\partial_Y a = \left\{ \frac{x \in Y}{x \not\asymp a} \right\}$ for an element $a$ of a poset $\mathfrak{A}$ and $Y \in \mathscr{P}\mathfrak{A}$.

DEFINITION 167. *Full star* of $a \in \mathfrak{A}$ is $\star a = \partial_{\mathfrak{A}} a$.

PROPOSITION 168. If $\mathfrak{A}$ is a meet-semilattice, then $\star$ is a straight monotone map.

PROOF. Monotonicity is obvious. Let $\star a \not\sqsubseteq \star(a \sqcap b)$. Then it exists $x \in \star a$ such that $x \notin \star(a \sqcap b)$. So $x \sqcap a \notin \star b$ but $x \sqcap a \in \star a$ and consequently $\star a \not\sqsubseteq \star b$. □

DEFINITION 169. A *separation subset* of a poset $\mathfrak{A}$ is such its subset $Y$ that

$$\forall a, b \in \mathfrak{A} : (\partial_Y a = \partial_Y b \Rightarrow a = b).$$

DEFINITION 170. I call separable such poset that $\star$ is an injection.

OBVIOUS 171. A poset is separable iff it has a separation subset.

DEFINITION 172. A poset $\mathfrak{A}$ has *disjunction property of Wallman* iff for any $a, b \in \mathfrak{A}$ either $b \sqsubseteq a$ or there exists a non-least element $c \sqsubseteq b$ such that $a \asymp c$.

THEOREM 173. For a meet-semilattice with least element the following statements are equivalent:FiXme: Condition to have least element seems superfluous.

1°. $\mathfrak{A}$ is separable.
2°. $\forall a, b \in \mathfrak{A} : (\star a \sqsubseteq \star b \Rightarrow a \sqsubseteq b)$.
3°. $\forall a, b \in \mathfrak{A} : (a \sqsubset b \Rightarrow \star a \sqsubset \star b)$.
4°. $\forall a, b \in \mathfrak{A} : (a \sqsubset b \Rightarrow \star a \neq \star b)$.
5°. $\forall a, b \in \mathfrak{A} : (a \sqsubset b \Rightarrow \star a \not\sqsupseteq \star b)$.
6°. $\forall a, b \in \mathfrak{A} : (\star a \sqsubseteq \star b \Rightarrow a \not\sqsupset b)$.
7°. $\mathfrak{A}$ conforms to Wallman's disjunction property.
8°. $\forall a, b \in \mathfrak{A} : (a \sqsubset b \Rightarrow \exists c \in \mathfrak{A} \setminus \{\bot\} : (c \asymp a \wedge c \sqsubseteq b))$.

PROOF.

1°⇔2°⇔3°⇔4°⇔5°⇔6°. By the above theorem.

8°⇒4°. Let property (8) hold. Let $a \sqsubset b$. Then it exists element $c \sqsubseteq b$ such that $c \neq \bot$ and $c \sqcap a = \bot$. But $c \sqcap b \neq \bot$. So $\star a \neq \star b$.

2°⇒7°. Let property (2) hold. Let $a \not\sqsubseteq b$. Then $\star a \not\sqsubseteq \star b$ that is it there exists $c \in \star a$ such that $c \notin \star b$, in other words $c \sqcap a \neq \bot$ and $c \sqcap b = \bot$. Let $d = c \sqcap a$. Then $d \sqsubseteq a$ and $d \neq \bot$ and $d \sqcap b = \bot$. So disjunction property of Wallman holds.

7°⇒8°. Obvious.