- Composition of morphisms is defined by the formula: $(B; C; g) \circ (A; B; f) = (A; C; g \circ f)$ where $g \circ f$ is function composition.

DEFINITION 148. The category **Rel** is:

- Objects are small sets.
- Morphisms from an object A to an object B are triples $(A; B; f)$ where f is a binary relation between A and B.
- Composition of morphisms is defined by the formula: $(B; C; g) \circ (A; B; f) = (A; C; g \circ f)$ where $g \circ f$ is relation composition.

I will denote $\mathrm{GR}(A; B; f) = f$ for any morphism $(A; B; f)$ of either **Set** or **Rel**.

I will denote $\langle f \rangle^* = \langle \mathrm{GR}\, f \rangle^*$ and $[f]^* = [\mathrm{GR}\, f]^*$ for any morphism $(A; B; f)$ of either **Set** or **Rel**.

DEFINITION 149. A morphism whose source is the same as destination is called *endomorphism*.

DEFINITION 150. FiXme: Definition of *subcategory* *Wide subcategory* of a category $(\mathcal{O}; \mathcal{M})$ is a category $(\mathcal{O}; \mathcal{M}')$ where $\mathcal{M} \subseteq \mathcal{M}'$ and the composition on $(\mathcal{O}; \mathcal{M}')$ is a restriction of composition of $(\mathcal{O}; \mathcal{M})$. (Similarly *wide sub-precategory* can be defined.)

2.3. Intro to group theory

DEFINITION 151. A semigroup is a pair of a set G and an associative binary operation on G.

DEFINITION 152. A group is a pair of a set G and a binary operation $\cdot$ on G such that:

1°. $(h \cdot g) \cdot f = h \cdot (g \cdot f)$ for every $f, g, h \in G$.
2°. There exists an element e (*identity*) of G such that $f \cdot e = e \cdot f = f$ for every $f \in G$.
3°. For every element f there exists an element f^{-1} such that $f \cdot f^{-1} = f^{-1} \cdot f = e$.

OBVIOUS 153. Every group is a semigroup.

PROPOSITION 154. In every group there exist exactly one identity element.

PROOF. If p and q are both identities, then $p = p \cdot q = q$. □

PROPOSITION 155. Every group element has exactly one inverse.

PROOF. Let p and q be both inverses of $f \in G$. Then $f \cdot p = p \cdot f = e$ and $f \cdot q = q \cdot f = e$. Then $p = p \cdot e = p \cdot f \cdot q = e \cdot q = q$. □

PROPOSITION 156. $(g \cdot f)^{-1} = f^{-1} \cdot g^{-1}$ for every group elements f and g.

PROOF. $(f^{-1} \cdot g^{-1}) \cdot (g \cdot f) = f^{-1} \cdot g^{-1} \cdot g \cdot f = f^{-1} \cdot e \cdot f = f^{-1} \cdot f = e$. Similarly $(g \cdot f) \cdot (f^{-1} \cdot g^{-1}) = e$. So $f^{-1} \cdot g^{-1}$ is the inverse of $g \cdot f$. □

DEFINITION 157. A *permutation group* on a set D is a group whose elements are functions on D and whose composition is function composition.

OBVIOUS 158. Elements of a permutation group are bijections.

DEFINITION 159. A *transitive* permutation group on a set D is such a permutation group G on D that for every $x, y \in D$ there exists $r \in G$ such that $y = r(x)$.

A groupoid with single (arbitrarily chosen) object corresponds to every group. The morphisms of this category are elements of the group and the composition of morphisms is the group operation.