On the other hand, $a^+ \sqcup b^+ \sqcup (a \sqcap b) = (a^+ \sqcup b^+ \sqcup a) \sqcap (a^+ \sqcup b^+ \sqcup b)$. Obviously $a^+ \sqcup b^+ \sqcup a = a^+ \sqcup b^+ \sqcup b = \top$. So $a^+ \sqcup b^+ \sqcup (a \sqcap b) \sqsupseteq \top$ and thus $a^+ \sqcup b^+ \sqsupseteq \top \setminus^* (a \sqcap b) = (a \sqcap b)^+$.

So $(a \sqcap b)^+ = a^+ \sqcup b^+$. □

2.2. Intro to category theory

I recall that this is a *very* basic introduction to category theory, I even do not define *functors* as they have no use in my theory.FiXme: Say instead: "no use in this book volume".

DEFINITION 137. A *directed multigraph* is:

1°. a set $\mathcal{O}$ (*vertices*);
2°. a set $\mathcal{M}$ (*edges*);
3°. functions Src and Dst (*source* and *destination*) from $\mathcal{M}$ to $\mathcal{O}$.

Note that in category theory vertices are called *objects* and edges are called *morphisms*.

DEFINITION 138. A *precategory* is a directed multigraph together with a partial binary operation $\circ$ on the set $\mathcal{M}$ such that $g \circ f$ is defined iff $\operatorname{Dst} f = \operatorname{Src} g$ (for every morphisms f and g) such that

1°. $\operatorname{Src}(g \circ f) = \operatorname{Src} f$ and $\operatorname{Dst}(g \circ f) = \operatorname{Dst} g$ whenever the composition $g \circ f$ of morphisms f and g is defined.
2°. $(h \circ g) \circ f = h \circ (g \circ f)$ whenever compositions in this equation are defined.

DEFINITION 139. The set $\operatorname{Mor}(A; B)$ (morphisms from an object A to an object B) is exactly morphisms which have A as the source and B as the destination.

DEFINITION 140. *Identity morphism* is such a morphism e that $e \circ f = f$ and $g \circ e = g$ whenever compositions in these formulas are defined.

DEFINITION 141. A *category* is a precategory with additional requirement that for every object X there exists identity morphism 1_X.

PROPOSITION 142. For every object X there exist no more than one identity morphism.

PROOF. Let p and q be both identity morphisms for a object X. Then $p = p \circ q = q$. □

DEFINITION 143. An *isomorphism* is such a morphism f of a category that there exists a morphism f^{-1} (*inverse* of f) such that $f \circ f^{-1} = 1_{\operatorname{Dst} f}$ and $f^{-1} \circ f = 1_{\operatorname{Src} f}$.

PROPOSITION 144. An isomorphism has exactly one inverse.

PROOF. Let g and h be both inverses of f. Then $h = h \circ 1_{\operatorname{Dst} f} = h \circ f \circ g = 1_{\operatorname{Src} f} \circ g = g$. □

DEFINITION 145. A *groupoid* is a category all of whose morphisms are isomorphisms.

Some important examples of categories:

EXERCISE 146. Prove that the below examples of categories are really categories.

DEFINITION 147. The category **Set** is:

- Objects are small sets.
- Morphisms from an object A to an object B are triples $(A; B; f)$ where f is a function from A to B.