2°.

$$\begin{aligned} a \sqcup (b \setminus^* a) &= \\ a \sqcup \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{b \sqsubseteq a \sqcup z} \right\} &= \\ \bigsqcap \left\{ \frac{a \sqcup z}{z \in \mathfrak{A}, b \sqsubseteq a \sqcup z} \right\} &= \\ a \sqcup b. \end{aligned}$$

3°. $b \sqcup (b \setminus^* a) = b \sqcup \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{b \sqsubseteq a \sqcup z} \right\} = \bigsqcap \left\{ \frac{b \sqcup z}{z \in \mathfrak{A}, b \sqsubseteq a \sqcup z} \right\} = b$.

4°. Obviously $(b \sqcup c) \setminus^* a \sqsupseteq b \setminus^* a$ and $(b \sqcup c) \setminus^* a \sqsupseteq c \setminus^* a$. Thus $(b \sqcup c) \setminus^* a \sqsupseteq (b \setminus^* a) \sqcup (c \setminus^* a)$. We have

$$\begin{aligned} (b \setminus^* a) \sqcup (c \setminus^* a) \sqcup a &= \\ ((b \setminus^* a) \sqcup a) \sqcup ((c \setminus^* a) \sqcup a) &= \\ (b \sqcup a) \sqcup (c \sqcup a) &= \\ a \sqcup b \sqcup c &\sqsupseteq \\ b \sqcup c. \end{aligned}$$

From this by definition of adjoints: $(b \setminus^* a) \sqcup (c \setminus^* a) \sqsupseteq (b \sqcup c) \setminus^* a$.

□

THEOREM 133. $\left(\bigsqcup S\right) \setminus^* a = \bigsqcup \left\{ \frac{x \setminus^* a}{x \in S} \right\}$ for all $a \in \mathfrak{A}$ and $S \in \mathscr{P}\mathfrak{A}$ where $\mathfrak{A}$ is a co-brouwerian lattice and $\bigsqcup S$ is defined.

PROOF. Because lower adjoint preserves all suprema. □

THEOREM 134. $(a \setminus^* b) \setminus^* c = a \setminus^* (b \sqcup c)$ for elements a, b, c of a complete co-brouwerian lattice.FiXme: can be generalized for a natural number of elements, using math induction.

PROOF. $a \setminus^* b = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}$. □

$(a \setminus^* b) \setminus^* c = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \setminus^* b \sqsubseteq c \sqcup z} \right\}$.

$a \setminus^* (b \sqcup c) = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup c \sqcup z} \right\}$.

It is left to prove $a \setminus^* b \sqsubseteq c \sqcup z \Leftrightarrow a \sqsubseteq b \sqcup c \sqcup z$. But this follows from corollary 126.

2.1.15. Dual pseudocomplement on co-Heyting lattices.

THEOREM 135. For co-Heyting algebras $\top \setminus^* b = b^+$.

PROOF.

$$\top \setminus^* b = \min \left\{ \frac{z \in \mathfrak{A}}{\top \sqsubseteq b \sqcup z} \right\} = \min \left\{ \frac{z \in \mathfrak{A}}{\top = b \sqcup z} \right\} = \min \left\{ \frac{z \in \mathfrak{A}}{b \equiv z} \right\} = b^+.$$

□

THEOREM 136. $(a \sqcap b)^+ = a^+ \sqcup b^+$ for every elements a, b of a co-Heyting algebra.

PROOF. $a \sqcup (a \sqcap b)^+ \sqsupseteq (a \sqcap b) \sqcup (a \sqcap b)^+ \sqsupseteq \top$. So $a \sqcup (a \sqcap b)^+ \sqsupseteq \top$; $(a \sqcap b)^+ \sqsupseteq 1 \setminus^* a = a^+$.

We have $(a \sqcap b)^+ \sqsupseteq a^+$. Similarly $(a \sqcap b)^+ \sqsupseteq b^+$. Thus $(a \sqcap b)^+ \sqsupseteq a^+ \sqcup b^+$.