PROOF.

$$(a \setminus^* b) \sqcup b =$$
$$\bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\} \sqcup b =$$
$$\bigsqcap \left\{ \frac{z \sqcup b}{z \in \mathfrak{A}, a \sqsubseteq b \sqcup z} \right\} =$$
$$\bigsqcap \left\{ \frac{t \in \mathfrak{A}}{t \sqsupseteq b, a \sqsubseteq t} \right\} =$$
$$a \sqcup b.$$

□

THEOREM 130. The following are equivalent for a complete lattice $\mathfrak{A}$:

1°. $\mathfrak{A}$ is meet infinite distributive.
2°. $\mathfrak{A}$ is a co-brouwerian lattice.
3°. $\mathfrak{A}$ is a co-Heyting lattice.
4°. $a \sqcup -$ has lower adjoint for every $a \in \mathfrak{A}$.

PROOF. □

2°⇔3°. Obvious (taking into account completeness of $\mathfrak{A}$).
4°⇒1°. Let $- \setminus^* a$ be the lower adjoint of $a \sqcup -$. Let $S \in \mathscr{P}\mathfrak{A}$. For every $y \in S$ we have $y \sqsupseteq (a \sqcup y) \setminus^* a$ by properties of Galois connections; consequently $y \sqsupseteq \left(\bigsqcap \langle a \sqcup \rangle^* S\right) \setminus^* a$; $\bigsqcap S \sqsupseteq \left(\bigsqcap \langle a \sqcup \rangle^* S\right) \setminus^* a$. So

$$a \sqcup \bigsqcap S \sqsupseteq \left(\left(\bigsqcap \langle a \sqcup \rangle^* S\right) \setminus^* a\right) \sqcup a \sqsupseteq \bigsqcap \langle a \sqcup \rangle^* S.$$

But $a \sqcup \bigsqcap S \sqsubseteq \bigsqcap \langle a \sqcup \rangle^* S$ is obvious.
1°⇒2°. Let $a \setminus^* b = \bigsqcap \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}$. To prove that $\mathfrak{A}$ is a co-brouwerian lattice it is enough to prove $a \sqsubseteq b \sqcup (a \setminus^* b)$. But it follows from the lemma.
2°⇒4°. $a \setminus^* b = \min \left\{ \frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z} \right\}$. So $a \sqcup -$ is the upper adjoint of $- \setminus^* a$.
1°⇒4°. Because $a \sqcup -$ preserves all meets.

COROLLARY 131. Co-brouwerian lattices are distributive.

The following theorem is essentially borrowed from [18]:

THEOREM 132. A lattice $\mathfrak{A}$ with least element $\bot$ is co-brouwerian with pseudodifference $\setminus^*$ iff $\setminus^*$ is a binary operation on $\mathfrak{A}$ satisfying the following identities:

1°. $a \setminus^* a = \bot$;
2°. $a \sqcup (b \setminus^* a) = a \sqcup b$;
3°. $b \sqcup (b \setminus^* a) = b$;
4°. $(b \sqcup c) \setminus^* a = (b \setminus^* a) \sqcup (c \setminus^* a)$.

PROOF.

⇐. We have

$$c \sqsupseteq b \setminus^* a \Rightarrow c \sqcup a \sqsupseteq a \sqcup (b \setminus^* a) = a \sqcup b \sqsupseteq b;$$
$$c \sqcup a \sqsupseteq b \Rightarrow c = c \sqcup (c \setminus^* a) \sqsupseteq (a \setminus^* a) \sqcup (c \setminus^* a) = (a \sqcup c) \setminus^* a \sqsupseteq b \setminus^* a.$$

So $c \sqsupseteq b \setminus^* a \Leftrightarrow c \sqcup a \sqsupseteq b$ that is $a \sqcup -$ is an upper adjoint of $- \setminus^* a$. By a theorem above our lattice is co-brouwerian. By another theorem above $\setminus^*$ is a pseudodifference.

⇒.

1°. Obvious.