PROOF. Because of duality it is enough to prove that $\bar{a}$ is pseudocomplement of a.

We need to prove $c \asymp a \Rightarrow c \sqsubseteq \bar{a}$ for every element c of our poset, and $\bar{a} \asymp a$. The second is obvious. Let's prove $c \asymp a \Rightarrow c \sqsubseteq \bar{a}$.

Really, let $c \asymp a$. Then $c \sqcap a = \bot$; $\bar{a} \sqcup (c \sqcap a) = \bar{a}$; $(\bar{a} \sqcup c) \sqcap (\bar{a} \sqcup a) = \bar{a}$; $\bar{a} \sqcup c = \bar{a}$; $c \sqsubseteq \bar{a}$. □

DEFINITION 119. Let $\mathfrak{A}$ be a join-semilattice. Let $a, b \in \mathfrak{A}$. *Pseudodifference* of a and b is

$$\min\left\{\frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z}\right\}.$$

If z is a pseudodifference of a and b we will denote $z = a \setminus^* b$.

REMARK 120. I do not require that a^* is undefined if there are no pseudocomplement of a and likewise for dual pseudocomplement and pseudodifference. In fact below I will define quasicomplement, dual quasicomplement, and quasidifference which generalize pseudo-* counterparts. I will denote a^* the more general case of quasicomplement than of pseudocomplement, and likewise for other notation.

OBVIOUS 121. Dual pseudocomplement is the dual of pseudocomplement.

DEFINITION 122. *Co-brouwerian lattice* is a lattice for which pseudodifference of any two its elements is defined.

PROPOSITION 123. Every non-empty co-brouwerian lattice $\mathfrak{A}$ has least element.

PROOF. Let a be an arbitrary lattice element. Then

$$a \setminus^* a = \min\left\{\frac{z \in \mathfrak{A}}{a \sqsubseteq a \sqcup z}\right\} = \min \mathfrak{A}.$$

So $\min \mathfrak{A}$ exists. □

DEFINITION 124. *Co-Heyting lattice* is co-brouwerian lattice with greatest element.

THEOREM 125. For a co-brouwerian lattice $a \sqcup -$ is an upper adjoint of $- \setminus^* a$ for every $a \in \mathfrak{A}$.

PROOF. $g(b) = \min\left\{\frac{x \in \mathfrak{A}}{a \sqcup x \sqsupseteq b}\right\} = b \setminus^* a$ exists for every $b \in \mathfrak{A}$ and thus is the lower adjoint of $a \sqcup -$. □

COROLLARY 126. $\forall a, x, y \in \mathfrak{A} : (x \setminus^* a \sqsubseteq y \Leftrightarrow x \sqsubseteq a \sqcup y)$ for a co-brouwerian lattice.

DEFINITION 127. Let $a, b \in \mathfrak{A}$ where $\mathfrak{A}$ is a complete lattice. *Quasidifference* $a \setminus^* b$ is defined by the formula:

$$a \setminus^* b = \bigsqcap\left\{\frac{z \in \mathfrak{A}}{a \sqsubseteq b \sqcup z}\right\}.$$

REMARK 128. A more detailed theory of quasidifference (as well as quasicomplement and dual quasicomplement) will be considered below.

LEMMA 129. $(a \setminus^* b) \sqcup b = a \sqcup b$ for elements a, b of a meet infinite distributive complete lattice.