Proof. Let $\mathfrak{A}$ be a bounded distributive lattice and $Z(\mathfrak{A})$ be its center. Let $a, b \in Z(\mathfrak{A})$. Consequently $\bar{a}, \bar{b} \in Z(\mathfrak{A})$. Then $\bar{a} \sqcup \bar{b}$ is the complement of $a \sqcap b$ because

$$(a \sqcap b) \sqcap (\bar{a} \sqcup \bar{b}) = (a \sqcap b \sqcap \bar{a}) \sqcup (a \sqcap b \sqcap \bar{b}) = \bot \sqcup \bot = \bot \quad \text{and}$$
$$(a \sqcap b) \sqcup (\bar{a} \sqcup \bar{b}) = (a \sqcup \bar{a} \sqcup \bar{b}) \sqcap (b \sqcup a \sqcup \bar{b}) = \top \sqcap \top = \top.$$

So $a \sqcap b$ is complemented. Similarly $a \sqcup b$ is complemented. □

Theorem 89. The center of a bounded distributive lattice constitutes a boolean lattice.

Proof. Because it is a distributive complemented lattice. □

2.1.10. Atoms of posets.

Definition 90. An atom of a poset is an element which has no non-least subelements.

Remark 91. This definition is valid even for posets without least element.

I will denote $\operatorname{atoms}^{\mathfrak{A}} a$ or just $(\operatorname{atoms} a)$ the set of atoms contained in an element a of a poset $\mathfrak{A}$. I will denote $\operatorname{atoms}^{\mathfrak{A}}$ the set of all atoms of a poset $\mathfrak{A}$.

Definition 92. A poset $\mathfrak{A}$ is called *atomic* iff $\operatorname{atoms} a \neq \emptyset$ for every non-least element a of the poset $\mathfrak{A}$.

Definition 93. *Atomistic poset* is such a poset that $a = \bigsqcup \operatorname{atoms} a$ for every non-least element a of this poset.

Obvious 94. Every atomistic poset is atomic.

Proposition 95. FiXme: It is also equivalent to $a \in \operatorname{atoms} B$.Let $\mathfrak{A}$ be a poset. If a is an atom of $\mathfrak{A}$ and $B \in \mathfrak{A}$ then $a \sqsubseteq B \Leftrightarrow a \not\asymp B$.

Proof. □

$\Rightarrow$. $a \sqsubseteq B \Rightarrow a \sqsubseteq a \wedge a \sqsubseteq B$, thus $a \not\asymp B$ because a is not least.

$\Leftarrow$. $a \not\asymp B$ implies existence of non-least element x such that $x \sqsubseteq B$ and $x \sqsubseteq a$. Because a is an atom, we have $x = a$. So $a \sqsubseteq B$.

Theorem 96. $\operatorname{atoms} \bigsqcap S = \bigcap \langle \operatorname{atoms} \rangle^* S$ whenever $\bigsqcap S$ is defined for every $S \in \mathscr{P}\mathfrak{A}$ where $\mathfrak{A}$ is a poset.

Proof. For any atom

$$\begin{aligned} c \in \operatorname{atoms} \bigsqcap S &\Leftrightarrow \\ c \sqsubseteq \bigsqcap S &\Leftrightarrow \\ \forall a \in S : c \sqsubseteq a &\Leftrightarrow \\ \forall a \in S : c \in \operatorname{atoms} a &\Leftrightarrow \\ c \in \bigcap \langle \operatorname{atoms} \rangle^* S&. \end{aligned}$$

□

Corollary 97. $\operatorname{atoms}(a \sqcap b) = \operatorname{atoms} a \cap \operatorname{atoms} b$ for an arbitrary meet-semilattice.

Theorem 98. A complete boolean lattice is atomic iff it is atomistic.

Proof.

$\Leftarrow$. Obvious.