2.1.1.1. *Intersecting and joining elements.* Let $\mathfrak{A}$ be a poset.

DEFINITION 23. Call elements a and b of $\mathfrak{A}$ *intersecting*, denoted $a \not\asymp b$, when there exists a non-least element c such that $c \sqsubseteq a \wedge c \sqsubseteq b$.

DEFINITION 24. $a \asymp b \overset{\text{def}}{=} \neg(a \not\asymp b)$.

OBVIOUS 25. $a_0 \not\asymp b_0 \wedge a_1 \sqsupseteq a_0 \wedge b_1 \sqsupseteq b_0 \Rightarrow a_1 \not\asymp b_1$.

DEFINITION 26. I call elements a and b of $\mathfrak{A}$ *joining* and denote $a \equiv b$ when there is no a non-greatest element c such that $c \sqsupseteq a \wedge c \sqsupseteq b$.

DEFINITION 27. $a \not\equiv b \overset{\text{def}}{=} \neg(a \equiv b)$.

OBVIOUS 28. Intersecting is the dual of non-joining.

OBVIOUS 29. $a_0 \equiv b_0 \wedge a_1 \sqsupseteq a_0 \wedge b_1 \sqsupseteq b_0 \Rightarrow a_1 \equiv b_1$.

2.1.2. Linear order.

DEFINITION 30. A poset $\mathfrak{A}$ is called *linearly ordered set* (or what is the same, *totally ordered set*) if $a \sqsupseteq b \vee b \sqsupseteq a$ for every $a, b \in \mathfrak{A}$.

EXAMPLE 31. The set of real numbers with the customary order is a linearly ordered set.

DEFINITION 32. A set $X \in \mathscr{P}\mathfrak{A}$ where $\mathfrak{A}$ is a poset is called *chain* if $\mathfrak{A}$ restricted to X is a total order.

2.1.3. Meets and joins. Let $\mathfrak{A}$ be a poset.

DEFINITION 33. Given a set $X \in \mathscr{P}\mathfrak{A}$ the *least element* (also called *minimum* and denoted $\min X$) of X us such $a \in X$ that $\forall x \in X : a \sqsubseteq x$.

Least element does not necessarily exists. But if it exists:

PROPOSITION 34. For a given $X \in \mathscr{P}\mathfrak{A}$ there exist no more than one least element.

PROOF. It follows from anti-symmetry. □

Greatest element is the dual of least element:

DEFINITION 35. Given a set $X \in \mathscr{P}\mathfrak{A}$ the *greatest element* (also called *maximum* and denoted $\max X$) of X us such $a \in X$ that $\forall x \in X : a \sqsupseteq x$.

REMARK 36. Least and greatest elements of a set X is a trivial generalization of the above defined least and greatest element for the entire poset.

DEFINITION 37.

- A *minimal* element of a set $X \in \mathscr{P}\mathfrak{A}$ is such $a \in \mathfrak{A}$ that $\nexists x \in X : (a \sqsupseteq x \wedge x \neq a)$.
- A *maximal* element of a set $X \in \mathscr{P}\mathfrak{A}$ is such $a \in \mathfrak{A}$ that $\nexists x \in X : (a \sqsubseteq x \wedge x \neq a)$.

REMARK 38. Minimal element is not the same as minimum, and maximal element is not the same as maximum.

OBVIOUS 39.

1°. The least element (if it exists) is a minimal element.
2°. The greatest element (if it exists) is a maximal element.

EXERCISE 40. Show that there may be more than one minimal and more than one maximal element for some poset.